Université de Carthage

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Préparée à

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En vue d’obtenir le Diplôme de

DOCTEUR

En

Technologies de l’Information et de la Communication

Par

Nazih HAJRI

Thème

Performance Analysis of Mobile-to-Mobile Communications over Hoyt Fading Channels

Soutenue à SUP’COM le 11 Mars 2011 devant le jury d’examen composé de :

Président : M. Ammar BOUALLEGUE - Professeur à L’ENIT
Rapporteurs :
  M. Mohamed-Slim ALOUINI - Professeur à KAUST
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Abstract

Mobile-to-mobile (M2M) communications, where both the transmitter and receiver are in motion, find many applications in ad-hoc wireless networks, intelligent transportation systems, and relay-based cellular networks. For both an effective M2M communication systems design and a related performance analysis, the appropriate propagation characteristics have to be taken into account. In this respect, the investigation of the error rate performance of the digital transmission has been widely studied for the case of M2M Rayleigh, Rice, and Nakagami-$m$ fading channels. Recently, and besides these most frequently used channels, the Hoyt fading is a widely accepted statistical model to characterize the short-term multipath effects, where the fading conditions are more severe than those of the Rayleigh case. Given the importance of the Hoyt fading channel, it is of a great interest, therefore, to study and analyze its impact on the performance of the wireless M2M communication systems. In this thesis, we contribute to the topic of the performance analysis of various digital transmission schemes over M2M Hoyt fading channels. In this context, our work can be divided into two essential parts. In the first one, we present a study on the performance analysis of the main digital angular modulation schemes under single Hoyt fading channels, taking into account the Doppler spread effects caused by the motion of the mobile transmitter and receiver, i.e, the case of a single Hoyt fading with a double-Doppler or M2M single Hoyt fading channels. In this framework, closed-form expressions for the bit error probability (BEP) performance of the differential phase-shift keying (PSK) modulation and frequency-shift keying (FSK) with limiter-discriminator integrator and differential detection schemes have been addressed under M2M Hoyt fading channels. In the second part, we introduce the double Hoyt fading model, which can be useful in the modeling of M2M fading channels, where the multipath propagation conditions are worse than those described by the double Rayleigh fading. This model assumes that the overall complex channel gain, between a mobile transmitter and a receiver, is modeled as the product of the gains of two statistically independent single Hoyt channels. By considering this M2M multipath fading distribution, the first and the second order statistics of the double Hoyt fading channels are first derived. As it is known, the second order statistics in terms of the level-crossing rate (or equivalently the frequency of outages) and average duration of fades (or equivalently the average outage duration) represent important commonly performance measures of wireless communication systems that are used to reflect the correlation properties of the fading channels and provide a dynamic representation of the system outage performance. Then, expressions for the main first and second order statistics of the corresponding channel capacity process are also investigated. Finally, the BEP of the digital modulated signals that are transmitted over slow and frequency flat double Hoyt fading channels is studied. In this case, a generic expression for the average BEP of coherent binary PSK, quadrature PSK, FSK, minimum-shift keying, and amplitude-shift keying modulation schemes is derived.

**Index Terms**— Mobile-to-mobile communications, bit error probability performance, Hoyt fading channels, mobile-to-mobile Hoyt fading channels, double Hoyt fading channels, probability of outage, average outage duration, frequency of outages, digital modulation schemes.
Preface

The work presented in this thesis has been published and presented in variety national and international conferences. Specifically, the thesis work has resulted in the following publications.

International Conferences


National Conferences

List of Acronyms

AAF Amplify and Forward
ABER Average Bit Error Rate
ACF Autocorrelation Function
ADF Average Duration of Fades
AMPS Advance Mobile Phone Service
AoA Angle of Arrival
AoD Angle of Departure
AoF Amount of Fading
ASEP Average Symbol Error Probability
ASK Amplitude Shift Keying
AWGN Additive White Gaussian Noise
BEP Bit Error Probability
BPSK Binary Phase-Shift-Keying
BS Base Station
CDF Cumulative Distribution Function
CDMA Code Division Multiple Access
CPM Continuous Phase Modulation
DPSK Differential Phase-Shift-Keying
DS Direct Sequence
DSRC Dedicated Short Range Communications
EGC Equal Gain Combining
ETACS European Total Access Communication System
F2M Fixed-to-Mobile
FM Frequency Modulation
FSK Frequency-Shift-Keying
GMSK Gaussian Minimum Shift Keying
IF Intermediate Frequency
IHVS Intelligent Highway Vehicular Systems
Nomenclature

iid independent and identically distributed
ISI InterSymbol Interference
LCR Level Crossing Rate
LD Limiter-Discriminator
LDI Limiter-Discriminator-Integrator
LOS Line-Of-Sight
LPNM $L_p$-Norm Method
M-DPSK $M$-ary Differential Phase-Shift Keying
M-QAM $M$-ary Quadrature Amplitude Modulation
M2M Mobile-to-Mobile
MEDS Method of Exact Doppler Spread
MIMO Multiple-Input Multiple-Output
MRC Maximum Ratio Combining
MSR Mobile Station Receiver
MSR Mobile Station Transmitter
MSEM Mean Square Error Method
MSK Minimum Shift Keying
PDF Probability Density Function
PSD Power Spectrum Density
PSK Phase-Shift-Keying
QPSK Quadrature Phase-Shift Keying
RS Relay Station
SIMO Single-Input Multiple-Output
SISO Single-Input Single-Output
SNR Signal-to-Noise Ratio

List of Symbols

$\alpha_{R,n}$ The AoA of the $n$th path measured with respect to the velocity vector $\vec{V}_R$
$\alpha_{T,n}$ The AoD of the $n$th path measured with respect to the velocity vector $\vec{V}_T$
$\alpha$ The ratio between the maximum Doppler frequencies $f_{R,\max}$ and $f_{T,\max}$
$\bar{\gamma}, \bar{\gamma}_s$ The average signal-to-noise ratio
$\beta_{ij}$ The negative curvature of the autocorrelation function $\Gamma_{\mu_i\mu_j}(t)$ ($i,j = 1,2$) at $\tau = 0$
\( \hat{\Gamma}_{\mu_i\mu_j}(\tau) \) The curvature of the autocorrelation function \( \Gamma_{\mu_i\mu_j}(t) \) \((i, j = 1, 2)\)

\( \Delta \eta \) The phase noise difference due to additive Gaussian noise

\( \Delta \phi \) The data phase difference

\( \Delta \theta \) The phase difference introduced by the Hoyt fading channel

\( \Delta \Omega \) The phase difference between two Hoyt faded signals perturbed by Gaussian noise

\( \Delta \Psi \) The overall phase difference at the output of a LDI circuit

\( \psi(t) \) The derivative of the overall phase \( \psi(t) \) with respect to the time \( t \)

\( \dot{\Xi}(t) \) The time derivative process of \( \Xi(t) \)

\( \dot{\Xi}^2(t) \) The time derivative of the process \( \Xi^2(t) \)

\( \dot{R}(t) \) The time derivative process of \( R(t) \)

\( \eta(t) \) The phase caused by additive Gaussian noise

\( \Gamma_{\mu_i\mu_i}(\tau) \) The temporal autocorrelation function of the process \( \mu_i(t) \) \((i = 1, 2)\)

\( \Gamma_{nn}(\tau) \) The temporal autocorrelation function of the process \( n_i(t) \) \((i = 1, 2)\)

\( \gamma(t), \gamma_s(t) \) The instantaneous signal-to-noise ratio

\( \Gamma_{\mu_i\mu_j}(\tau) \) The temporal autocorrelation function of the process \( \mu_{ij} \) \((i = 1, 2)\)

\( \Gamma_{g_s_t}(\tau) \) The temporal autocorrelation function of the process \( g_s(t) \) \((i = 1, 2)\)

\( \Lambda \) The ratio between the variances \( \sigma_1^2 \) and \( \sigma_n^2 \)

\( \mu_{ij}(t) \) Zero-mean Gaussian process \((i, j = 1, 2)\)

\( \mu_1(t) \) The Hoyt channel gain process

\( \Omega(t) \) The phase caused by the Hoyt fading plus additive Gaussian noise

\( \mathcal{N} \) The average number of FM clicks

\( \mathcal{P}_b \) The average BEP performance in double Hoyt fading channels

\( \vec{V}_R \) The velocity vector due to the motion of the receiver

\( \vec{V}_T \) The velocity vector due to the motion of the transmitter

\( \phi(t) \) The filtered signal phase after FM modulation

\( \psi(t) \) The overall phase at the output of the limiter circuit

\( \rho_{\mu_{11} + n} \) The normalized ACF of the process \((\mu_{11}(t) + n_1(t))\)

\( \rho_{\tau_i} \) The normalized ACF of the process \(\mu_{1i}(t) \) \((i = 1, 2)\)

\( \Sigma \) The covariance matrix of the vector process \((x_1, y_1, x_2, y_2)^t\)

\( \sigma^2 \) The mean power of the Hoyt fading process \( R(t) \)

\( \Sigma^{-1} \) The inverse matrix of the covariance matrix \( \Sigma \)
The average power of the additive Gaussian noise $n(t)$

The variance of the single Rayleigh fading process $R_{D_i}(t)$ ($i = 1, 2$)

The reduced variance of the process $\mu_{1i}(t)$ ($i = 1, 2$)

The variance of the process $\mu_{ij}(t)$ ($i, j = 1, 2$)

The variance of the process $g_{si}(t)$ ($i = 1, 2$)

The phase process of the double Hoyt fading channel

The FSK data phase

The random phase

The phase of the deterministic process $\bar{\mu}_{1i}(t)$ ($i = 1, 2$)

The phase of the deterministic process $\bar{\mu}_{ij}(t)$ ($i, j = 1, 2$)

The random phase around the mobile station receiver

The random phase around the mobile station transmitter

The complex double Hoyt channel gain process

The single Hoyt channel phase process ($i = 1, 2$)

The deterministic process corresponding to the process $\mu_{ij}(t)$ ($i, j = 1, 2$)

The double Hoyt fading process

The double Hoyt channel power gain

The noise component defined relatively to the coordinate system that rotate with $\phi_1$ ($i = 1, 2$)

The filtered carrier amplitude

The random amplitude around the mobile station transmitter

The equivalent noise bandwidth

The binary data sequence

The random amplitude around the mobile station receiver

The bandwidth-time product coefficient

The normalized time varying capacity process of double Hoyt fading channels

The amplitude of the $n$th propagation path

The gains of the deterministic process $\bar{\mu}_{1i}(t)$ ($i = 1, 2$)

The gains of the deterministic process $\bar{\mu}_{ij}(t)$ ($i, j = 1, 2$)

The joint amplitude caused by the interaction of the transmitter and receiver scatterers

The Hoyt faded sinusoid signals perturbed by Gaussian noise
\( e_0(t) \) The FSK signal at the output of the IF Gaussian filter

\( e_1(t) \) The signal at the output of the limiter circuit

\( E_b \) The average energy per bit

\( E_b/N_0 \) The bit-energy-to-noise ratio

\( F_C(c) \) The cumulative distribution function of the channel capacity \( C(t) \)

\( f_{\text{max},i} \) The maximum Doppler frequency corresponding to the process \( \mu_{1i} \) \( (i = 1, 2) \)

\( f_{ij,n} \) The discrete Doppler frequency of the deterministic process \( \tilde{\mu}_{ij}(t) \) \( (i, j = 1, 2) \)

\( f_{R,\text{max}} \) The maximum Doppler frequency generated by the motion of the mobile receiver

\( f_{R,n}^i \) The discrete Doppler frequency of the process \( \tilde{\mu}_{1i}(t) \) \( (i = 1, 2) \) caused by the motion of the receiver

\( f_{T,\text{max}} \) The maximum Doppler frequency generated by the motion of the mobile transmitter

\( f_{T,n}^i \) The discrete Doppler frequency of the process \( \tilde{\mu}_{1i}(t) \) \( (i = 1, 2) \) caused by the motion of the transmitter

\( g_D(t) \) The general complex M2M double ring channel gain process

\( g_d(t) \) The complex M2M double ring channel gain process

\( g_s(t) \) The complex M2M single Rayleigh channel gain process

\( h \) The FSK modulation index

\( H(f) \) The low-pass transfer function of a Gaussian filter

\( J \) The jacobian determinant

\( M \) The number of scatterers located in the mobile station transmitter end ring

\( m \) The fading severity parameter of Nakagami-\( m \) channel

\( N \) The number of scatterers located in the mobile station receiver end ring

\( n(t) \) The additive Gaussian noise process

\( N(t_0 - T, t_0) \) The click noise component generated in the interval \([t_0 - T, t_0]\)

\( N_0 \) The one-sided power spectral density of the additive white Gaussian noise

\( n_1(t) \) The in-phase zero-mean Gaussian noise component

\( n_2(t) \) The quadrature zero-mean Gaussian noise component

\( N_{\Xi}(r) \) The level-crossing rate of the double Hoyt fading process \( \Xi(t) \)

\( N_{C}(c) \) The level-crossing rate of the channel capacity \( C(t) \)

\( N_i \) The number of sinusoids used for the generation of process \( \tilde{\mu}_{1i}(t) \) \( (i = 1, 2) \)

\( N_{ij} \) The number of sinusoids used for the process \( \tilde{\mu}_{ij}(t) \) \( (i, j = 1, 2) \)

\( N_{R_s}(r) \) The level-crossing rate of the M2M single Rayleigh fading process \( R_s(t) \)
Nomenclature

$N_R(r)$ The level-crossing rate of the Hoyt fading process $R(t)$

$p$ The parameter that depends on the different coherent modulation schemes

$p_\theta(\theta)$ The probability density function of the Hoyt phase process $\theta(t)$

$p_\Xi(z)$ The probability density function of the process $\Xi(t)$

$p_C(c)$ The probability density function of the channel capacity $C(t)$

$P_E$ The bit error probability

$P_{E}(M)$ The conditional probability of error given the transmission of a mark

$P_{E}(S)$ The conditional probability of error given the transmission of a space

$p_m$ The probability of a mark in the information signal

$p_R(z)$ The probability density function of the Hoyt fading process $R(t)$

$p_{\Delta \eta}(\varphi)$ The probability density function of the phase difference $\Delta \eta$

$p_{\Delta \Omega}(\varphi)$ The probability density function of the phase difference $\Delta \Omega$

$p_{\Delta \vartheta}(\varphi)$ The probability density function of the phase difference $\Delta \vartheta$

$p_{\gamma_\alpha}(\beta)$ The probability density function of the instantaneous SNR $\gamma_\alpha(t)$

$p_{\psi_1, \psi_2}(\cdot, \cdot)$ The joint PDF of the random phases $\psi_1$ and $\psi_2$

$p_{\Theta}(\theta)$ The probability density function of the phase process $\Theta(t)$

$p_{\vartheta_1, \vartheta_2}(\cdot, \cdot)$ The joint probability density function of the phase process $\vartheta_1(t)$ and $\vartheta_2(t)$

$p_{\vartheta_i}(\theta)$ The probability density function of the phase process $\vartheta_i(t)$ ($i = 1, 2$)

$P_{\Xi_{-}}(r)$ The cumulative distribution function of the double Hoyt fading process $\Xi(t)$

$p_{\Xi, \hat{\Xi}}(\cdot, \cdot)$ The joint probability density function of the process $\Xi(t)$ and $\hat{\Xi}(t)$

$p_{\Xi^2, \hat{\Xi}^2}(\cdot, \cdot)$ The joint probability density function of the process $\Xi^2(t)$ and $\hat{\Xi}^2(t)$

$p_{CC}(\cdot, \cdot)$ The joint probability density function of the process $C(t)$ and $\hat{C}(t)$

$p_{R, \hat{R}}(\cdot, \cdot)$ The joint PDF of the process $R(t)$ and its time derivative $\hat{R}(t)$

$P_{R_{-}}(r)$ The probability that the process $R(t)$ is found below the level $r$

$p_{R_1, R_2}(\cdot, \cdot)$ The joint probability density function of the process $R_1(t)$ and $R_2(t)$

$p_{R_1, R_2, \psi_1, \psi_2}(\cdot, \cdot, \cdot, \cdot)$ The joint PDF of the random variables $R_1$, $R_2$, $\psi_1$, and $\psi_2$

$p_{R_D}(z)$ The probability density function of the double Rayleigh fading process $R_D(t)$

$p_{R, \hat{R}_i}(\cdot, \cdot)$ The joint probability density function of the process $R_i(t)$ and $\hat{R}_i(t)$ ($i = 1, 2$)

$p_{R_i}(z)$ The probability density function of the Hoyt fading process $R_i(t)$ ($i = 1, 2$)

$q, q_1, q_2$ The Hoyt fading parameter
Nomenclature

$q_0$ The reduced Hoyt fading parameter

$R(t)$, $R_i(t)$ The single Hoyt fading process ($i = 1, 2$)

$R_D(t)$ The double Rayleigh fading process

$R_s(t)$ The M2M single Rayleigh fading process

$R_{D_i}(t)$ The single Rayleigh fading process ($i = 1, 2$)

$s(t)$ The signal at the output of an angular modulator

$s_0(t)$ The noise corrupted fading signal at the output of the rectangular bandpass filter

$s_1(t)$ The product of the real part of the signal $s_0(t)$ with its real delayed version $s_0(t - T)$

$s_d(t)$ The filtered output signal of a DPSK receiver

$s_r(t)$ The FSK received signal after transmission over Hoyt fading channel

$s_t(t)$ The FSK transmitted signal

$S_{\mu_i}(f)$ The Doppler power spectral density of the process $\mu_i(t)$ ($i = 1, 2$)

$S_{\mu_{ij}}(f)$ The Doppler power spectral density of the process $\mu_{ij}(t)$ ($i,j = 1, 2$)

$S_{g_s}(f)$ The Doppler power spectral density of the process $g_s(t)$ ($i = 1, 2$

$T$ The one bit duration

$T_C(c)$ The average duration of fades of the channel capacity $C(t)$

$T_{\Xi}(r)$ The average duration of fades of the double Hoyt fading process $\Xi(t)$

$T_R(r)$ The average duration of fades of the Hoyt fading process $R(t)$

$T_{R_s}(r)$ The average duration of fades of the M2M single Rayleigh fading process $R_s(t)$

$V_R$ The speed of the mobile Receiver

$V_T$ The speed of the mobile transmitter

Operators

$(\cdot)^H$ The transpose operator

$\det$ The determinant operator

$\Re \{\cdot\}$ The reel operator

$\text{Var} \{\cdot\}$ The variance operator

$E \{\cdot\}$ The expected value operator

$\text{Prob}(X(t) > x)$ The probability that the variable $X(t)$ verifies $X(t) > x$

Special Functions

$Q(\cdot)$ The Gaussian $Q$-function

$F(\cdot)$ The hypergeometric function
$K_0(\cdot)$ The zeroth-order modified Bessel function of the second kind
erfc(\cdot) The complementary error function
$K(\cdot)$ The complete elliptic integral of the first kind
$I_0(\cdot)$ The zeroth-order modified Bessel function of the first kind
$J_0(\cdot)$ The zeroth-order Bessel function of the first kind
$W_{-1/2,0}(\cdot)$ The Whittaker’s function
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Mobile-to-mobile (M2M) communications have recently received much attention due to some new applications, such as wireless mobile ad-hoc networks [Tonguz 2006], relay-based cellular networks [Adinoyi 2007], and dedicated short range communications (DSRC) for intelligent transportation systems (e.g., IEEE 802.11p standard) [Report 2007]. In contrast to conventional fixed-to-mobile (F2M) cellular radio systems, in M2M communications both the transmitter and receiver are in motion and equipped with low elevation antennas. Hence, the double mobility, combined with the low elevation antennas at both the two end mobile units, make the fading distribution and the Doppler power spectral density (PSD) of M2M channels have statistical characteristics that are different from those that are corresponding to the case of conventional cellular systems. Therefore, current results and models that are developed for the F2M cellular radio systems may not be directly applicable to the M2M communication systems, and new channel models are required for the newly corresponding wireless communications. Early studies of single-input single-output (SISO) M2M Rayleigh fading channels have been reported by Akki et al. in [Akki 1986]. They proposed the first so-called Akki and Haber’s M2M channel model also known as the single Rayleigh double-Doppler model [Kovacs 2002a]. They showed that the received envelope of M2M channels is Rayleigh faded under non line-of-sight (LOS) conditions, but the statistical properties of the corresponding Doppler spectrum differ from conventional F2M cellular radio channels. Indeed, the resulting spectrum of M2M channels is a convolution of two classical Doppler spectrums and, therefore, the result is a double-Doppler PSD. More recently, Kovacs et al. [Kovacs 2002a, Kovacs 2002b] studied the characteristics of the M2M communications in a forest environment and a short distance built-up environment. In this case, they proposed the so-called cascaded or double Rayleigh multipath fading model which appears as a more realistic and appropriate channel model for modeling the underlying M2M propagation scenarios. This M2M channel model assumed that the overall complex channel gain between the mobile transmitter and receiver is obtained as the product of the gains of two statistically independent Rayleigh fading channels. Therefore, the received envelope of corresponding M2M channels is double Rayleigh faded. However, there is no difference between
1. Introduction

the Doppler PSD functions of single and double Rayleigh fading models and, therefore, double Rayleigh channels are also characterized by the so-called M2M double-Doppler PSD.

Being relatively a new area of research, M2M communication channels pose numerous interesting research problems. Specifically, for both an effective M2M communication systems design and a related performance analysis, the appropriate propagation characteristics have to be taken into account. In this respect, several papers dealing with the performance investigation of the M2M radio links have appeared in the literature. One of the earliest works has been reported in [Akki 1994a], where the impact of the double-Doppler spectrum of the M2M single Rayleigh fading channels on the BEP performance of the FSK modulation has been studied. In [Hagras 2001], we have presented a comparison study on the performance of the Gaussian minimum-shift keying (GMSK) modulation with the center sampling differential detection and center sampling double differential detection schemes over the M2M Rayleigh fading channels taking into account the random frequency shifts and the co-channel interference. The effect of the double-Doppler PSD on the BEP performance of the differential quadrature PSK modulation scheme over frequency flat M2M single Rayleigh channels has been studied in [Ali 2007].

In the context of double scattering channels, an expression for the symbol error probability of some common communication schemes over the so-called double Rayleigh fading channels has been derived [Salo 2006a]. Concerning also this M2M channel model, Uysal et al. [Uysal 2006] presented an exact expression for the pair wise error probability of the space-time trellis codes. The moment generating function and average BEP of $M$-ary phase-shift keying ($M$-PSK) modulation with a maximum ratio combining (MRC) diversity over double Rician fading channels have been presented in [Wongtrairat 2008]. The performance analysis problem of M2M radio links has also been addressed in [Wongtrairat 2009] under the double Nakagami-$m$ fading distribution and for the case of the MRC diversity based receivers.

Besides the channel models described above, the Nakagami-$q$ (Hoyt) distribution is a widely accepted statistical model for characterizing the short-term multipath effects [Hoyt 1947]. This fading channel, originally introduced by Nakagami [Nakagami 1960] and Hoyt [Hoyt 1947], includes the one-sided Gaussian and Rayleigh distributions as special cases. In addition, this generic model is known to approximate the Nakagami-$m$ model if the severity fading parameter $m$ is between 0.5 and 1 [Nakagami 1960]. Given the importance of the Hoyt channel distribution in a statistical modeling of the short-term multipath fading, it is of a great interest, therefore, to study and analyze its impact on the performance of the wireless M2M communication systems. To the best of our knowledge, there exists no investigation on this topic so far. The present thesis work aims at studying and analyzing the performance of the wireless M2M communication
systems over Hoyt multipath fading channels.

Namely, the objective of this thesis is to contribute to the topic of performance analysis of various digital modulation schemes commonly used in wireless communication systems over the M2M Hoyt fading channels.

1.1 Thesis Objectives and contributions

This thesis focuses on the performance analysis of the digital M2M communications over Hoyt fading channels. Namely, the obtained performance results will be applied to specific M2M communication scenarios, where the multipath effects are modeled by the Hoyt fading distribution, i.e., the so-called M2M Hoyt fading channels. In this framework, the present thesis embraces, in essence, two principal parts. In the first part, we focus on the analysis of the error rate performance of wireless communications over a single Hoyt fading with a double Doppler PSD, i.e. the M2M single Hoyt fading channel. In the second one, we concentrate on studying and analyzing the performance of the digital wireless M2M communications over the double Hoyt fading channels. To the best of our knowledge, the double Hoyt fading channel, where the overall complex channel gain between the mobile transmitter and the receiver can be modeled as the product of the gains of two statistically independent single Hoyt channels, is new and has not been proposed in the literature so far. In addition, this channel distribution can be used in the modeling of the M2M fading channels, where the fading conditions are worse than those described by the double Rayleigh fading case. Moreover, the double Hoyt fading model is generic and includes the double fading distributions described by the combinations Rayleigh×Hoyt, double Rayleigh, Rayleigh×one-sided Gaussian, and the double one-sided Gaussian as special cases. Hence, the obtained results will be generic and valid for all these double multipath fading models as special cases.

1.1.1 Performance analysis of the M2M communications over the single Hoyt double-Doppler multipath fading channels

The principal objectives of the first part of this thesis, which considers the performance analysis of wireless M2M communications over a single Hoyt double-Doppler multipath propagation environment, are as follows:

- To derive analytical expressions for the BEP performance of various digital modulation schemes over a single Hoyt fading channel, taking into account the Doppler spread effects caused by the motion of the mobile transmitter and receiver.
1.1. Thesis Objectives and contributions

- To check the validity of the obtained theoretical results by using computer simulations.
- To emphasize the applicability of the obtained theoretical results, for the BEP performance, on the M2M communications. Specifically, we try to evaluate the error rate performance of the M2M communication systems over Hoyt fading channels, i.e., the case of the M2M single Hoyt fading channels. This study can be done by just replacing, in the derived BEP expression, the generic autocorrelation function (ACF) by that proposed for the M2M fading channel, i.e, the double-Doppler ACF. [Akki 1986].

Regarding this part, a variety of new contributions are obtained. Namely, the main investigated results pertaining to the performance analysis of the wireless communications over the M2M single Hoyt fading channels can be summarized as follows:

1. Derivation of the BEP performance of the FSK modulation with limiter-discriminator-integrator (LDI) detection over the Hoyt fading channels. In this study, the performance analysis of the narrowband digital FSK modulation with the LDI detection over Hoyt mobile fading channels is presented. Closed-form expressions are derived, separately, for the probability density function (PDF) of the phase angle between two Hoyt fading vectors and for the phase noise due to an additive Gaussian noise. By assuming the statistical independence between the two random phase processes, an expression for the resultant PDF, of the overall phase difference caused by the Hoyt fading plus the Gaussian noise, is deduced by a convolution operation. In addition to the resultant PDF, an expression for the average number of the FM clicks occurring at the output of a digital frequency modulation (FM) receiver, is also derived. Based on all these investigated quantities, a formula for the BEP performance of the FSK transmission over Hoyt fading channels and perturbed by an additive Gaussian noise is obtained. However, the resulting BEP formula demands tedious numerical integrations, due to a convolution operation, and is valid only for small values of signal-to-noise ratio (SNR). Furthermore, the obtained BEP expression takes into account the Doppler spread effects but it can only be applied on the Jakes’ Doppler model.

2. Derivation of the distribution of the phase difference between two modulated Hoyt processes perturbed by the Gaussian Noise. To avoid the drawback of the obtained BEP results mentioned above, the derivation of the resultant PDF of the overall phase difference between two modulated Hoyt faded signals and perturbed by the Gaussian noise, is approached from a different point of view. In this step, we derive a general closed-form expression for...
the PDF of the overall phase difference between two phase or frequency modulated signals transmitted over the Hoyt fading channels and perturbed by the Gaussian noise. The validity of the obtained PDF expression is demonstrated by simulations for the case of a M2M single Hoyt fading channel. The newly derived PDF is general and can be applied in the performance analysis of the wireless M2M communication systems, considering a Hoyt multipath propagation environment, for various digital transmission techniques. Namely, the BEP performance of the FSK and the DPSK schemes, where the decision variable needed for the demodulator is the differential phase quantity, can be analyzed.

3. *The BEP analysis of the DPSK modulation schemes over Hoyt fading channels.* Based on the newly derived PDF expression for the overall phase difference, the BEP performance analysis of the DPSK modulation over Hoyt fading channels, considering the Doppler spread effects, that are caused by the motion of the transmitter and receiver, is analyzed and investigated. The obtained theoretical quantities are fully validated by comparing them with the results obtained from the computer simulations, considering a M2M single Hoyt fading channel.

4. *The BEP analysis of the FSK with the LDI and the differential detection schemes over Hoyt fading channels.* The obtained PDF expression for the phase difference between two Hoyt faded signals and perturbed by the Gaussian noise, is also applied in studying the BEP performance of narrowband digital FSK receives with an LDI detection over the M2M single Hoyt fading channels. In addition, the BEP performance of one-bit delay differential detection of narrowband FSK signals transmitted over M2M Hoyt fading channels and corrupted by AWGN is investigated.

The publications that are associated to this part are the following:


1.1. Thesis Objectives and contributions


1.1.2 Performance analysis of the M2M communications over the double Hoyt fading channels

The double Hoyt fading model is new and, to the best of our knowledge, this thesis is the first to investigate a study on this channel model so far. Indeed, there are not any published works so far on the statistical characterization of the double Hoyt fading channels as well as the investigation of the error rate performance of the digital transmissions over such scattering cascaded channels. Hence, in order to contribute to the error rate performance of the M2M communications over double Hoyt fading channels, the investigation of the basic statistical properties of these fading channels is also mostly required. Therefore, the main objectives of the second part of this thesis can be classified as follows:

- To study the statistical properties of the double Hoyt fading channels.

- To investigate the main statistics of the capacity of the double Hoyt fading channels, based on the use of the obtained statistics for the double Hoyt fading processes.

- To study and analyze the performance of the wireless M2M communications over a double Hoyt fading channel perturbed by the additive white Gaussian noise (AWGN). Specifically, we try to derive an analytical expression for the BEP performance of the most commonly used digital modulation schemes over a such M2M multipath fading channel.

- To check the validity of the obtained theoretical results by using the computer simulations.

To achieve all the specified objectives of the second part of our thesis work, numerous important results have been obtained. These results are:
1. Derivation of the statistical properties of the double Hoyt fading channels. In this step, analytical expressions for the mean value, variance, PDF, cumulative distribution function (CDF), level-crossing rate (LCR), and average duration of fades (ADF) of double Hoyt fading processes are derived. As it is known, the CDF is useful for studying the outage probability, which is a performance measure commonly used in wireless communications. In addition, the LCR (or equivalently the frequency of outages) and ADF (or equivalently the average outage duration) represent important commonly performance measures of wireless communication systems that are used to reflect the correlation properties (thus the second order statistics) of fading channels and provide a dynamic representation of the system outage performance [Guizani 2004]. Furthermore, in this last step, an expression for the PDF of the corresponding channel phase is provided. Moreover, the validity of all the derived quantities is confirmed by computer simulations.

2. Derivation of the statistical properties of the capacity of double Hoyt fading channels. Apart from the amplitude and phase statistics, and with the growing interest in high spectrally efficient systems, the investigation and analysis of the dynamical behavior of the fading channel capacity is becoming an important topic of research. In this step, analytical expressions for the PDF and CDF of the instantaneous capacity process are derived. Furthermore, the second order statistics, in the form of the LCR and ADF, are as well provided. The correctness of all the derived quantities is confirmed by computer simulations.

3. The BEP analysis of the digital modulation schemes over slowly varying frequency flat double Hoyt fading channels. A performance analysis of digital modulation schemes over a slowly varying frequency flat double Hoyt fading channel perturbed by an AWGN is presented. First, an expression for the PDF of the SNR is derived. Then, based upon this PDF a general expression for the average BEP of the most common digital coherent modulation schemes is investigated. Namely, the BEP performance of coherent binary PSK (BPSK), quadrature phase-shift keying (QPSK), FSK, minimum-shift keying (MSK), and amplitude-shift keying (ASK) modulation schemes is presented. In addition, the validity of the obtained theoretical results is demonstrated by comparing them with results obtained from computer simulations, for some of the modulation schemes used in the analysis.

The publications that are associated to this part are the following:
1.2 Thesis Organization

For the purpose of presenting all the above mentioned results, we have chosen to divide the thesis into 8 chapters.

Chapter 2. This chapter introduces the Hoyt multipath fading model and presents an overview on the wireless M2M communications channels.

Chapter 3. This chapter presents all the details of the derivation of an analytical expression for the BEP performance of a narrowband digital FSK modulation scheme with an LDI detection over the Hoyt mobile fading channels, taking into account the Doppler spread of the Jakes’ model.

Chapter 4. In this chapter, we look first at the derivation of a closed-form expression for the PDF of the phase difference between two phase or frequency modulated signals that are transmitted over Hoyt fading channels and perturbed by the correlated Gaussian noise. Then, the comparison of the obtained theoretical results with corresponding simulation ones, considering a M2M single Hoyt fading channel, are presented.

Chapter 5. The BEP performance analysis of the digital wireless communications over Hoyt multipath fading channels is presented in this chapter. Namely, the BEP of DPSK
detection schemes together with that of FSK with LDI and differential detections, considering a M2M single Hoyt multipath fading channel, are provided.

**Chapter 6.** This chapter starts by studying the first and second order statistics of the double Hoyt fading channels. Indeed, expressions for the mean value, variance, envelope PDF, phase PDF, LCR, and ADF of the double Hoyt channels are derived. Then, we concentrate, in this chapter, on the derivation of the first and second order statistics of the corresponding channel capacity processes. In this case, analytical expressions for the PDF, CDF, LCR, and ADF of the capacity of double Hoyt channels are presented. Finally, the validity of all the derived quantities for the fading and channel capacity processes is confirmed by computer simulations.

**Chapter 7.** This chapter considers the performance analysis of the M2M radio communications over double Hoyt fading channels perturbed by AWGN. First, by assuming that the double Hoyt propagation channel is slowly varying and having the frequency-fat fading characteristics, the PDF of the instantaneous SNR per bit in the double Hoyt fading channels is derived. Then, an expression for the BEP performance is investigated for several modulation schemes commonly used in wireless communication systems. Finally, the validity of the theoretical results is also checked by means of computer simulations, for some of the modulation schemes used in the analysis.

**Chapter 8.** This chapter concludes the dissertation. In addition, we outline some possible future research directions in this chapter.
Chapter 2

Literature review

The purpose of this chapter is to review, in Section 2.1, the principal characteristics of the Hoyt fading channels. Then, Section 2.2 presents an overview on the M2M communication channels. Finally, Section 2.3 concludes the chapter.

2.1 An overview of the Nakagami-\(q\) (Hoyt) fading model

2.1.1 Related studies

The Nakagami-\(q\) (Hoyt) fading distribution has been originally introduced by [Hoyt 1947] as a distribution of the modulus of a complex Gaussian random variable whose components are uncorrelated with zero-mean and unequal variances. Then, Nakagami et al. [Nakagami 1960] investigated this model as an approximation for the Nakagami-\(m\) fading distribution in the range of fading that extends from the one-sided Gaussian model to the Rayleigh model. Hence, the Nakagami-\(q\) (Hoyt) model has been sometimes referred to as the Nakagami-Hoyt distribution [Crepeau 1992]. In [Hoyt 1947], the first order statistics of the Hoyt fading channel have been derived. Specifically, closed-form expressions for the envelope PDF and phase PDF of Hoyt fading channels have been presented. Originally, the Hoyt fading model has been used by [Nakagami 1960] and [Chytil 1967] for modeling radio channels subject to strong ionospheric scintillation, such as satellite links. Recently, the model is being more and more useful in channel modeling of multipath fading, where the fading conditions are more severe than those of the classical Rayleigh case. For instance, in [Youssef 2005a], the crossing statistics of the phase processes and random FM noise of the Nakagami-\(q\) channel have been studied. In [Youssef 2005b], the second order statistics of the Hoyt fading channel in terms of the LCR and ADF have been presented. Besides these statistics, Youssef et al. [Youssef 2005b] showed that the Hoyt fading model can be applied to realistic mobile satellite channel, in the case of an environment with heavy shadowing. In dealing with the statistical characterization of the Hoyt fading channels and given the explosion of interest and research, that is taking place in the area of diversity reception based on the MRC and equal gain combining (EGC) techniques [Simon 2005], exact
2.1. An overview of the Nakagami-$q$ (Hoyt) fading model

Form expressions for the LCR and ADF of MRC and EGC diversity systems in the Hoyt fading environment have been presented in [Fraidenraich 2005]. Recently, in [Stefanović 2007], a closed-form expression for the envelope LCR of a cosine signal with a constant amplitude interfered by the Nakagami-$q$ process has been derived. More recently, Ricardo et al. [Ricardo 2008] investigated an exact closed-form and general expressions for the marginal and joint moments as well as for the correlation coefficient of the instantaneous powers of two Hoyt signals.

Being relatively an important general multipath fading model, which has been frequently used in the channel modeling of short-term multipath effects, the Hoyt fading channel has recently gained a widespread attention in the performance analysis of wireless communication systems. For example, in [Simon 1998, Simon 2005], a unified approach to evaluate the error rate performance of the digital communication systems operating over a generalized fading channel, especially a Hoyt fading channel, has been presented. This approach relies on employing alternative representations of classic functions arising in the error probability analysis of the digital communication systems (e.g., the Gaussian $Q$-function and the Marcum $Q$-function [Gradshteyn 1994]) in such a manner that the resulting expressions for various performance measures such as average bit or symbol error rate are in a form that is rarely more complicated than a single integral with infinite limits and an integrand composed of elementary (e.g., exponential and trigonometric [Gradshteyn 1994]) functions. Similarly, a unified framework for the error performance of $M$-ary quadrature amplitude modulation ($M$-QAM), employing $L$-branch EGC and operating over the Hoyt fading channels, has been provided in [Karagiannidis 2005]. By considering $M$-ary continuous phase modulations (CPM) systems with a differential phase detection and a MRC, Korn et al. [Korn 2001] analyzed the BEP performance of the underlying diversity systems in various fading channels, especially for the case of a slowly varying Hoyt fading channel. In [Liu 2006], a study dealing with the BEP performance of the asynchronous binary direct sequence (DS) code division multiple access (CDMA) systems operating over frequency flat Hoyt fading channels has been presented. In addition, exact closed-form expressions for the moments of the SNR at the output of the EGC over Hoyt fading channels have been derived by Zogas et al. [Zogas 2005]. In a recent work, Iskander et al. [Iskander 2008] investigated the derivation of an expression for the average symbol error probability (ASEP) of an EGC receiver. However, their presented results have been limited to dual-diversity case only. To avoid the drawback of the above obtained results, Baid et al. [Baid 2008] studied the general case by considering an arbitrary number of diversity branches and analyzed the performance of a predetection EGC receiver in independent Hoyt fading channels. Hence, mathematical expressions for the average output SNR and the average bit error rate (ABER) have been provided.
2.1. An overview of the Nakagami-q (Hoyt) fading model

for coherent BPSK, FSK, DPSK, and noncoherent FSK modulation schemes. More recently, Radaydeh et al. [Radaydeh 2008b] derived exact form expressions for the average error performance of $M$-ary orthogonal signals (such as $M$-FSK) with noncoherent EGC diversity systems in non-identical generalized Rayleigh, Rician, Nakagami-$m$, and implicitly Nakagami-$q$ fading channels. Within the framework of the error rate performance of the digital transmission over the Hoyt multipath fading channels, Radaydeh et al. derived, in [Radaydeh 2007], exact form expressions for the ASEP of SISO systems using $M$-ary modulation schemes. Excited by the importance and the much research activity on systems with multiple antennas at the transmitter or the receiver or at both, closed-form expressions for the ASEP of binary and $M$-ary modulated signals transmitted over single-input multiple-output (SIMO) and multiple-input multiple-output (MIMO) Hoyt diversity systems have been derived in [Duong 2007]. To reflect a real noncoherent receiver with a diversity reception for binary orthogonal signals transmitted over a Hoyt fading channel, Radaydeh et al. [Radaydeh 2008a] assumed that the multipath fading model is a non-identical Nakagami-$q$ fading channel. In this case, the average fading powers and the fading parameters take arbitrary values among diversity branches. Based on this assumption, the BEP performance of the obtained realistic diversity receiver operating over Hoyt fading channels has been presented, in terms of elementary functions. From the literature above, it is worth noting that the Hoyt fading channel is being used more frequently in the channel modelling and performance analysis related to mobile radio communications. In the following, we present the main statistical properties of this fading channel model.

2.1.2 An elementary description

In general, the Hoyt channel gain is often modeled, in the equivalent complex baseband representation, by a zero-mean complex Gaussian process $\mu_1(t)$ given by [Hoyt 1947]

$$\mu_1(t) = \mu_{11}(t) + j\mu_{12}(t)$$

(2.1)

where $\mu_{11}(t)$ and $\mu_{12}(t)$ are uncorrelated zero-mean Gaussian processes with different variances $\sigma_{11}^2$ and $\sigma_{12}^2$, respectively. The complex channel gain $\mu_1(t)$ can also be expressed as

$$\mu_1(t) = R(t) \exp[j\vartheta(t)]$$

(2.2)
2.1. An overview of the Nakagami-$q$ (Hoyt) fading model

where the instantaneous amplitude $R(t)$ stands for the Hoyt fading process. This amplitude is obtained as the modulus of the complex random process $\mu_1(t)$ according to

$$R(t) = |\mu_{11}(t) + j\mu_{12}(t)| = \sqrt{\mu_{11}^2(t) + \mu_{12}^2(t)}. \quad (2.3)$$

where $|\cdot|$ stands for the modulus operator. In (2.2), the process $\vartheta(t)$ represents the Hoyt channel phase. This channel phase process can be obtained from the in-phase and quadrature components $\mu_{11}(t)$ and $\mu_{12}(t)$, respectively, as follows

$$\vartheta(t) = \tan^{-1} \left( \frac{\mu_{12}(t)}{\mu_{11}(t)} \right). \quad (2.4)$$

As any other multipath fading model, the Hoyt fading distribution is characterized by its first and second order statistics. Namely, the PDFs of the envelope and phase constitute the most important first order statistics that are generally used in the description of a multipath fluctuations behavior. For the second order statistics, the ACF, LCR, and ADF distributions represent the necessary statistical quantities that are commonly used in the description of the dynamical behavior of the fading channels. Expressions for all these statistics (of the first and second order), in the case of Hoyt fading channels, have been proposed in the literature and will be presented below.

2.1.3 PDF of the envelope and phase processes

In this section, we present the PDF of the Hoyt process $R(t)$ and that of the corresponding phase process $\vartheta(t)$. Following [Hoyt 1947], the envelope PDF $p_R(z)$ of the Hoyt fading process $R(t)$ is given by [Hoyt 1947]

$$p_R(z) = \begin{cases} 
\frac{(1+q^2)}{\sigma^2 q} z \exp \left[ -\frac{z^2}{4\sigma^2} \left( 1+\frac{q^2}{q} \right) \right] \cdot I_0 \left[ \frac{z^2}{4\sigma^2} \left( 1-\frac{q^4}{q^2} \right) \right], & z \geq 0 \\
0, & z < 0
\end{cases} \quad (2.5)$$

where $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind [Gradshteyn 1994], and $q$ stands for the Hoyt fading parameter defined by [Nakagami 1960]

$$q = \frac{\sigma_{12}}{\sigma_{11}}, \quad \text{with } 0 \leq q \leq 1. \quad (2.6)$$
2.1. An overview of the Nakagami-\(q\) (Hoyt) fading model

In (2.5), the quantity \(\sigma^2\) stands for the mean power of the Hoyt fading process \(R(t)\) and is given by

\[
\sigma^2 = \sigma_{11}^2 + \sigma_{12}^2. \tag{2.7}
\]

It should be mentioned that for the special case corresponding to \(q = 1\), i.e., the case of Rayleigh fading channel, the PDF \(p_R(z)\) of the Hoyt process \(R(t)\) is reduced to the Rayleigh distribution given by [Rice 1944, Rice 1945]

\[
p_R(z) = \begin{cases} \frac{z}{\sigma_{11}^2} \exp \left[ -\frac{z^2}{4\sigma_{11}^2} \right], & z \geq 0 \\ 0, & z < 0. \end{cases} \tag{2.8}
\]

Besides the Hoyt fading distribution extends the Rayleigh fading model, i.e., \(q = 1\), it also contains the one-sided Gaussian distribution that corresponds to the case of \(q = 0\). Indeed, we first start by writing (2.5) as follows

\[
p_R(z) = \frac{2}{\sqrt{2\pi} \sigma} \sqrt{\frac{1 + q^2}{1 - q^2}} \exp \left[ -\frac{z^2 (1 + q^2)}{2\sigma^2} \right] \sqrt{2\pi X} \exp [-X] I_0 [X], \quad z \geq 0. \tag{2.9}
\]

where the quantity \(X\) is obtained to be

\[
X = \frac{z^2}{2\sigma^2} \left[ \frac{1 - q^4}{q^2} \right]. \tag{2.10}
\]

Then, by letting \(q = 0\), i.e., \(X \to \infty\), the quantity \(\sqrt{2\pi X} \exp [-X] I_0 [X]\) tends to 1, and thus (2.9) leads to the PDF of the one-sided Gaussian distribution given by

\[
p_R(z) = \frac{2}{\sqrt{2\pi} \sigma_{11}} \exp \left[ -\frac{z^2}{2\sigma_{11}^2} \right], \quad z \geq 0. \tag{2.11}
\]

In addition to the Rayleigh and one-sided Gaussian fading distributions, the Hoyt fading model can approximates the Nakagami-\(m\) distribution when the fading severity parameter \(m\) is in the range \(0.5 \leq m \leq 1\) [Nakagami 1960]. The influence of the fading severity parameter \(q\) on the PDF \(p_R(z)\) of the Hoyt fading process \(R(t)\) is shown, in Figure 2.1, for the mean-square value \(\sigma^2 = 1\).

Concerning the distribution of the Hoyt fading phase \(\theta(t)\), it is obtained to be [Hoyt 1947]

\[
p_\theta(\theta) = \frac{q}{2\pi (q^2 \cos^2(\theta) + \sin^2(\theta))}, \quad -\pi \leq \theta < \pi. \tag{2.12}
\]
2.1. An overview of the Nakagami-q (Hoyt) fading model

Figure 2.1: The PDF $p_R(z)$ of the Hoyt fading process $R(t)$ for different values of the fading parameter $q$.

Figure 2.2: The PDF $p_\theta(\theta)$ of the Hoyt phase process $\theta(t)$ for different values of the fading parameter $q$. 
2.1. An overview of the Nakagami-$q$ (Hoyt) fading model

As expected, when $q = 1$, i.e., the case of Rayleigh fading channels, (2.12) yields to a uniform distribution over $[-\pi, \pi)$. The effect of the parameter $q$ on the phase PDF $p_\theta(\theta)$ of the Hoyt fading channel can be studied from Figure 2.2.

2.1.4 Second order statistics

2.1.4.1 Autocorrelation function

An important statistical characteristic of the in-phase and quadrature random processes $\mu_{11}(t)$ and $\mu_{12}(t)$, respectively, is their ACF. Indeed, with a simple calculation and analysis of the ACF of these processes, we can determine how quickly each corresponding random process changes with respect to the time function. The ACF of the zero-mean Gaussian process $\mu_{1i}(t)$ ($i = 1, 2$) represents the expected value of the product of the signal $\mu_{1i}(t)$ at a time $t$ with a time-shifted version of itself at a time $t + \tau$, i.e., $\mu_{1i}(t + \tau)$. Thus, the corresponding ACF $\Gamma_{\mu_{1i}\mu_{1i}}(\tau)$ is defined by

$$\Gamma_{\mu_{1i}\mu_{1i}}(\tau) = E\{\mu_{1i}^*(t)\mu_{1i}(t + \tau)\}, \quad i = 1, 2 \tag{2.13}$$

where the superscripted asterisk $*$ denotes the complex conjugation, and $E\{\cdot\}$ stands for the expected value operator. Based on the ACFs $\Gamma_{\mu_{11}\mu_{11}}(\tau)$ and $\Gamma_{\mu_{12}\mu_{12}}(\tau)$ of the processes $\mu_{11}(t)$ and $\mu_{12}(t)$, respectively, the ACF of the complex zero-mean Gaussian process $\mu_{1}(t)$ in (2.1) can be written as [Papoulis 2002]

$$\Gamma_{\mu_{1}\mu_{1}}(\tau) = \Gamma_{\mu_{11}\mu_{11}}(\tau) + \Gamma_{\mu_{12}\mu_{12}}(\tau). \tag{2.14}$$

An important use of the ACF resides in the fact that the PSD can be directly obtained from it. The general relation between the PSD $S_{\mu_{1i}\mu_{1i}}(f)$ of the process $\mu_{1i}(t)$ ($i = 1, 2$) and the ACF $\Gamma_{\mu_{1i}\mu_{1i}}(\tau)$ is known as the Wiener-Khinchine relationship [CouchII 2001]. Following Wiener’s theorem, the PSD $S_{\mu_{1i}\mu_{1i}}(f)$ ($i = 1, 2$) of the random process $\mu_{1i}(t)$ is the Fourier transform of the corresponding ACF $\Gamma_{\mu_{1i}\mu_{1i}}(\tau)$. Thus, $S_{\mu_{1i}\mu_{1i}}(f)$ can be written as

$$S_{\mu_{1i}\mu_{1i}}(f) = \int_{-\infty}^{\infty} \Gamma_{\mu_{1i}\mu_{1i}}(\tau)e^{-j2\pi f \tau}d\tau, \quad i = 1, 2. \tag{2.15}$$

The shape of the PSD of the complex channel gain $\mu_{1}(t)$ is identical to the Doppler PSD, which is obtained from both the power of all electromagnetic waves arriving at the receiver antenna and the distribution of the angles of arrival. In addition to that, the radiation pattern of the receiving antenna has a decisive influence on the shape of the Doppler PSD.
2.1. An overview of the Nakagami-\(q\) (Hoyt) fading model

![ACF for Jakes’ model, \(\Gamma_{\mu_{11}\mu_{11}}(\tau)\) of the Gaussian process \(\mu_{11}(t)\) for the Jakes’ model and different values of the fading parameter \(q\).](image)

-\(q = 0.2\)
-\(-q = 0.6\)
-\(-q = 1.0\)

\(\sigma^2 = 1\)
\(f_{\text{max}} = 92\ \text{Hz}\)

Figure 2.3: The ACF \(\Gamma_{\mu_{11}\mu_{11}}(\tau)\) of the Gaussian process \(\mu_{11}(t)\) for the Jakes’ model and different values of the fading parameter \(q\).

[Clarke 1968, Jakes 1993, Pätzold 2002]. By assuming omnidirectional antennas, we can then easily calculate the Doppler PSD \(S_{\mu_{1}\mu_{1}}(f)\) of the scattered component \(\mu_{1}(t)\). In this case and following [Clarke 1968, Jakes 1993], the Doppler PSD \(S_{\mu_{1}\mu_{1}}(f)\) of the complex process \(\mu_{1}(t)\) can be expressed as

\[
S_{\mu_{1}\mu_{1}}(f) = S_{\mu_{11}\mu_{11}}(f) + S_{\mu_{12}\mu_{12}}(f)
\]  

(2.16)

where \(S_{\mu_{11}\mu_{11}}(f)\) corresponds to the Jakes Doppler PSD, which is given by [Clarke 1968]

\[
S_{\mu_{11}\mu_{11}}(f) = \frac{\sigma^2_{\mu_{11}}}{\pi f_{\text{max},i} \sqrt{1-(f/f_{\text{max},i})}}, \quad |f| \leq f_{\text{max},i}
\]

\[
0, \quad |f| > f_{\text{max},i}
\]

(2.17)

In (2.17), \(f_{\text{max},i}\) denotes the maximum Doppler frequency corresponding to the Gaussian process \(\mu_{1i}(t)\) \((i = 1, 2)\). The inverse Fourier transform of (2.17) results in the ACF \(\Gamma_{\mu_{1i}\mu_{1i}}(\tau)\) of the process \(\mu_{1i}(t)\) for the Jakes’ model, which is given by [Jakes 1993]

\[
\Gamma_{\mu_{1i}\mu_{1i}}(\tau) = \sigma^2_{\mu_{1i}} J_0(2\pi f_{\text{max},i}\tau), \quad i = 1, 2
\]

(2.18)

where \(J_0(\cdot)\) denotes the zeroth-order Bessel function of the first kind [Gradshteyn 1994]. The shape of the ACF \(\Gamma_{\mu_{11}\mu_{11}}(\tau)\) of the process \(\mu_{11}(t)\) for the Jakes’ model is presented together
2.1. An overview of the Nakagami-\( q \) (Hoyt) fading model

with the corresponding Doppler PSD function \( S_{\mu_1,\mu_1}(f) \), in Figures 2.3 and 2.4, respectively, for \( \sigma^2 = 1, f_{\text{max}} = 92 \, \text{Hz} \), and different values of the Hoyt fading parameter \( q \). Observing Figure 2.4, the Jakes Doppler PSD is symmetrical about zero within the frequency interval limited to the range \( |f| \leq f_{\text{max}} \). This shape is generally known as the U-shape spectrum [Jakes 1993]. On the other hand, there are poles at \( f = \pm f_{\text{max}} \). It should also be noted, from this figure, that the greatest Doppler spectrum is obtained for \( q = 1 \), i.e., the case of the Rayleigh fading channel. Hence, it can be concluded that the Hoyt fading model can be used in modelling of multipath fading channels, where the propagation conditions are more severe than those corresponding to the Rayleigh fading case.

### 2.1.4.2 Level crossing rate

The LCR of the Hoyt distributed random variable \( R(t) \), denoted here by \( N_R(r) \), describes how often the Hoyt process \( R(t) \) crosses a given level \( r \) from up to down (or from down to up) per time unit. According to [Rice 1944, Rice 1945], the LCR \( N_R(r) \) of the Hoyt fading process \( R(t) \) can be obtained by solving the following integral

\[
N_R(r) = \int_0^\infty \hat{z} p_{RR}(r, \hat{z}) \, d\hat{z}
\]  

(2.19)
2.1. An overview of the Nakagami-\(q\) (Hoyt) fading model

Figure 2.5: The normalized LCR \(N_R(r)/f_{\text{max}1}\) of the Hoyt fading process \(R(t)\) for different values of the fading parameter \(q\).

where \(p_{RR}(z,\dot{z})\) denotes the joint PDF of the process \(R(t)\) and its time derivative \(\dot{R}(t) = dR(t)/dt\) at the same time instant. Analytical expression for the LCR \(N_R(r)\) has recently been derived by Youssef et al. [Youssef 2005b] and is given by

\[
N_R(r) = \frac{r(1+q^2)}{(2\pi)^{3/2}q^2} \int_0^{2\pi} \left[ -\frac{r^2(1+q^2)}{2q^2\sigma^2} \left( q^2 \cos^2(\theta) + \sin^2(\theta) \right) \right] \exp \left[ -\frac{r^2(1+q^2)}{2q^2\sigma^2} \right] d\theta \times \sqrt{\beta_{12} + (\beta_{11} - \beta_{12})\cos^2(\theta)} d\theta \tag{2.20}
\]

where the quantity \(\beta_{1i} (i = 1, 2)\) stands for the negative curvature of the ACF \(\Gamma_{\mu_{1i}\mu_{1i}}(t)\) of the process \(\mu_{1i}(t)\) at \(t = 0\) according to

\[
\beta_{1i} = -\frac{d^2}{dt^2} \Gamma_{\mu_{1i}\mu_{1i}}(t) \bigg|_{t=0} = \ddot{\Gamma}_{\mu_{1i}\mu_{1i}}(0), \quad i = 1, 2. \tag{2.21}
\]

It can be mentioned that in the special case corresponding to \(q = 1\), i.e., the Rayleigh fading case, the quantities \(\beta_{11}\) and \(\sigma_{11}^2\) are equal to \(\beta_{12}\) and \(\sigma_{12}^2\), respectively, and thus (2.20) reduces to the well known LCR expression of the Rayleigh fading channel given by [Jakes 1993]

\[
N_R(r) = \sqrt{\frac{\beta_{11}}{2\pi}} \cdot \frac{r}{\sigma_{11}^2} \exp \left[ -\frac{r^2}{2\sigma_{11}^2} \right], \quad r \geq 0. \tag{2.22}
\]
By assuming Jakes Doppler PSD for the Gaussian processes $\mu_{11}(t)$ and $\mu_{12}(t)$, the corresponding quantities $\beta_{11}$ and $\beta_{12}$ can then be written as

$$\beta_{11} = \frac{2}{1 + q^2} (\pi \sigma f_{\text{max}1})^2$$  \hspace{1cm} (2.23)$$

and

$$\beta_{12} = \frac{2q^2}{1 + q^2} (\pi \sigma f_{\text{max}2})^2,$$  \hspace{1cm} (2.24)$$

respectively. Using these quantities, the normalized LCR $N_R(r)/f_{\text{max}1}$ of the Hoyt fading channel is depicted, in Figure 2.5, for $\sigma^2 = 1$, $f_{\text{max}1} = f_{\text{max}2} = 92$ Hz, and different values of the Hoyt fading parameter $q$. Thereby, it should be noted from this figure that an increase or a decrease of the fading severity parameter $q$ extremely influences the shape of the LCR $N_R(r)$.

### 2.1.4.3 Average duration of fades

The ADF of the Hoyt distributed random variable $R(t)$, denoted here by $T_{R-}(r)$, represents the expected value for the length of the time intervals in which the Hoyt fading process $R(t)$ is below a given level $r$. Following [Jakes 1993], the ADF $T_{R-}(r)$ is defined by

$$T_{R-}(r) = \frac{P_{R-}(r)}{N_r(r)}$$  \hspace{1cm} (2.25)$$

where $P_{R-}(r)$ represents the probability that the process $R(t)$ is found below the level $r$ according to

$$P_{R-}(r) = \int_0^r p_R(z) \, dz. \hspace{1cm} (2.26)$$

This quantity is also known as the CDF. By substituting (2.5) in (2.26), an expression for the probability $P_{R-}(r)$ is obtained to be as follows

$$P_{R-}(r) = \int_0^r \frac{(1 + q^2) z}{q^2} \exp \left[ - \frac{z^2}{4\sigma^2} \left( \frac{(1 + q^2)^2}{q^2} \right) \right] \cdot I_0 \left[ \frac{z^2}{4\sigma^2} \left( \frac{1 - q^4}{q^2} \right) \right] \, dz. \hspace{1cm} (2.27)$$

Finally, substituting (2.27) and (2.20) in (2.25) results in a formula for the ADF $T_{R-}(r)$ of the Hoyt fading process $R(t)$. By the way of illustration, Figure 2.6 shows an example for the normalized ADF $T_{R-}(r) \cdot f_{\text{max}1}$ of the Hoyt fading process $R(t)$ for $\sigma^2 = 1$, $f_{\text{max}1} = f_{\text{max}2} = 92$ Hz and different values of the Hoyt fading parameter $q$. 
2.2 An overview of the M2M communication channels

This section is started by a brief review of certain previous studies and results for wireless M2M communication channels. Then, a short overview of the principal models and statistical characteristics of the M2M Rayleigh fading channels is presented. Namely, the M2M single Rayleigh and double Rayleigh fading channel models are reviewed.

2.2.1 Related studies

As it has been mentioned, M2M channels, where the mobile station transmitter (MS\textsubscript{T}) and mobile station receiver (MS\textsubscript{R}) can be on the move, are being more and more useful in a wide range of applications for future wireless communication systems. Indeed, they have first been applied in ad-hoc wireless networks, which represent decentralized networks that can be installed on-the-fly in environments without the need of inflexible fixed base station (BS) infrastructure, i.e., stationary BS [Toh 2001]. This type of networks is generally considered in military arenas, given its simplicity of installation in battlefield environments [Plesse 2004, Baugé 2004, Tonguz 2006]. Another potential application of the M2M communications is best exemplified by intelligent transportation systems. Specifically, the M2M communications have been used in intelligent highway vehicular systems (IHVS) for automated traffic control on highways, road safety, and

Figure 2.6: The normalized ADF \( T_{R \left( r \right)} \cdot f_{\text{max},1} \) of the Hoyt fading process \( R(t) \) for different values of the fading parameter \( q \).

In addition, these types of communication systems are recently becoming more and more useful in the relay-based networks [Kovacs 2002b, Kovacs 2002a, Patel 2006]. In these networks, the transmission between the MS\textsubscript{T} and MS\textsubscript{R} is achieved through an intermediate transceiver unit called a relay station (RS) or a repeater, which is placed between the source and the destination to achieve an end-to-end communication. This repeater, which can be fixed or mobile, amplify and retransmit the received signal from the MS\textsubscript{T} to the MS\textsubscript{R}. Recently, the relay-based networks are becoming a hot research topic in the new wireless generation systems.

This type of networks has been proposed under different names such as “cooperation diversity” [Laneman 2001, Sendonaris 2003a, Sendonaris 2003b, Laneman 2004], “virtual antenna arrays” [Dohler 2003], or “multihop networks” [Verdone 1997, Pabst 2004, Boyer 2004].

Given the importance of M2M radio communications, which represent an emergency technology that has encompassed a wide range of important applications, they are being a relatively new area of research and pose numerous interesting research problems [Akki 1986, Akki 1994b, Vatalaro 1997, Kovacs 2002a, Kovacs 2002b, Maurer 2002, Acosta 2004, Linnartz 1996, Wang 2005]. So far, several papers have first focused on modeling the M2M radio channels by proposing the acceptable M2M channel models and studying their basic statistical properties. For example, the main statistical properties of the SISO M2M single Rayleigh fading channel have been studied by Akki et al. in [Akki 1986, Akki 1994b]. Specifically, in [Akki 1986], closed-form expressions for the received envelope PDF, phase PDF, and temporal ACF together with its corresponding Doppler PSD for the Akki and Haber’s M2M model, i.e., the M2M single Rayleigh fading model, have been derived. Concerning also this M2M channel model, analytical expressions for the LCR and ADF have been investigated [Akki 1994b]. In addition, Akki et al. [Akki 1994b] derived the PDF of random FM, power spectrum of random FM, and expected number of crossings of the random phase and random FM over the M2M single Rayleigh fading channels. The Akki and Haber’s M2M model has, then, been extended by Vatalaro et al. [Vatalaro 1997] for the case of three-dimensional scattering environments and they have studied the behavior of the corresponding Doppler PSD. Recently, in [Kovacs 2002a, Kovacs 2002b], a propagation model, known as the cascaded or double Rayleigh multipath fading model, has been proposed for a channel modelling of more realistic and appropriate M2M propagation scenarios. Potential applications of the double fading channel are best exemplified by direct M2M communications, where the transmitter and the receiver are separated by a large distance; or where diffracting wedges, such as street corners, exist in the radio propagation path [Salo 2006b]. This fading model, where the overall complex channel gain be-
2.2. An overview of the M2M communication channels

tween the MS\textsubscript{T} and MS\textsubscript{R} is obtained as the product of the gains of two Rayleigh fading distributions, has originally been identified in indoor [Honcharenko 1995] and micro-cellular radio propagation environments [Erceg 1997]. Nowadays, the channel measurements for outdoor-to-indoor narrowband M2M communication scenarios have been studied by Kovacs in [Kovacs 2002a]. Within the framework of the M2M channels modeling, Maurer et al. [Maurer 2002] presented channel measurements for narrowband outdoor-to-outdoor M2M communications. Recently and due to increasing demands for high-speed communications, future M2M communication systems are operating in larger bandwidths than today’s systems. Therefore, measurements for wideband M2M communication channels have been reported in [Acosta 2004]. Besides the Rayleigh fading model, the Rician scattering is of a great importance in the modeling of fading channels with LOS impairments. Hence, a statistical model for a narrowband M2M channel, taking into account the LOS component and using the Rice’s sum-of-sinusoids concept [Rice 1944, Rice 1945], has been presented in [Linnartz 1996, Wang 2005]. Given the importance of MIMO diversity channels, which offer larger gains in capacity over SISO channels, the design of MIMO reference models for M2M communications has gained nowadays a widespread attention and has been the topic of several papers [Pätzold 2005a, Pätzold 2008, Chelli 2007, Cheng 2008]. For example, Pätzold et al. [Pätzold 2005a, Pätzold 2008] derived a stochastic reference model for MIMO narrowband M2M channels. The obtained MIMO M2M model has been deduced from the geometrical two-ring scattering model under the assumption that both the MS\textsubscript{T} and MS\textsubscript{R} are surrounded by an infinite number of local scatterers. More recently, Ma et al. [Ma 2007] extended the narrowband two-ring MIMO reference channel model of Pätzold et al. [Pätzold 2005a, Pätzold 2008] to a MIMO wideband channel model for the M2M communications. For the case of MIMO M2M Rician fading channels, a generic and an adaptive geometrical-based stochastic reference model has recently been proposed by Cheng et al. [Cheng 2008].

Besides the modeling of M2M fading channels as well as the investigation of their statistical properties, the development of their proper channel simulators also plays a major role in the design and performance evaluation of related wireless communication systems. Nowadays, the topic of simulating M2M channels has become more and more prevalent and several papers on this topic have appeared in the literature. For example, Wang et al. [Wang 2002] applied a method based on approximating the continuous Doppler spectrum by a discrete line spectrum in order to simulate the M2M single fading channels. However, this method required a numerical integration of the Doppler spectrum, and thus, it has not always been suitable for the real time hardware channel emulation or software simulation. Therefore, Patel et al. [Patel 2003, Patel 2005] extended at first the single-ring model of Wang et al. [Wang 2002] to a double-ring
model, which has assumed a ring of uniformly spaced scatterers around both the MS\textsubscript{T} and MS\textsubscript{R}. Then, using the obtained double-ring model and based upon the Rice’s sum-of-sinusoids concept [Rice 1944, Rice 1945], Patel et al. modified the deterministic method of exact Doppler spread (MEDS), which has been proposed in [Pätzold 2002] for simulating the conventional cellular channel. Thereafter, they used their modified MEDS method in order to simulate the M2M single Rayleigh fading channel. Within the framework of the simulation of SISO M2M fading channels, in [Hajri 2005], the mean square error method (MSEM) and L\textsubscript{p}-norm method (LPNM), that have been applied by [Pätzold 2002] in the simulation of the conventional F2M channel, have first been extended and, then, applied in the simulation of the so-called M2M single Rayleigh fading channels. In addition, the deterministic MEDS method has been applied in the design of the M2M double Rayleigh fading simulators [Hajri 2005]. Recently, Zajié et al. [Zajié 2006] proposed a new general statistical sum-of-sinusoids model for the M2M single Rayleigh fading channels. This new proposed model has been proved to converge faster than previous existing simulation models [Patel 2005]. More recently, the simulation results for MIMO M2M channels, using the Rice’s sum-of-sinusoids concept, have been presented by Pätzold et al. in [Pätzold 2005b, Pätzold 2008].

In the following, we present a brief summary on the principal statistics of the two reference models that have been proposed for SISO M2M radio propagation channels. Namely, the M2M single Rayleigh and double Rayleigh fading channel models are discussed below.

2.2.2 The M2M single Rayleigh model

2.2.2.1 Akki and Haber’s model

In [Akki 1986], Akki et al. proposed the first statistical distribution for modeling the M2M radio propagation channels. They considered that the M2M propagation path between the MS\textsubscript{T} and MS\textsubscript{R}, which are moving with constant velocities $V_T = \|\overrightarrow{V_T}\|$ and $V_R = \|\overrightarrow{V_R}\|$, respectively, is obstructed by buildings, obstacles, etc., resulting in non LOS propagation conditions. In addition, they assumed that the scatterers around the mobile transmitter and receiver terminals are considered grouped. Hence, there are $N$ distinct single propagation paths between the MS\textsubscript{T} and MS\textsubscript{R}. This scattering model is also known as single-ring model [Kovacs 2002b]. Figure 2.7 illustrates a typical propagation path between the MS\textsubscript{T} and MS\textsubscript{R} over single-ring M2M scattering channels. The transmitted signal from the MS\textsubscript{T} undergoes a reflection, diffuse scattering, and a diffraction on encountering the obstacles. As a result, for a narrowband transmission over the frequency flat fading conditions, the received baseband complex channel gain at the MS\textsubscript{R}
is composed of the superposition of replicas due to scattering and the reflection phenomena. Hence, the corresponding complex channel gain process can be expressed as [Akki 1986]

\[ g_s(t) = g_{s1}(t) + jg_{s2}(t) = \lim_{N \to \infty} \sum_{n=1}^{N} c_n \exp \left[ j \left( 2\pi \left\{ f_{T,\text{max}} \cos(\alpha_{T,n}) + f_{R,\text{max}} \cos(\alpha_{R,n}) \right\} t + \theta_n \right) \right] \] (2.28)

where \( c_n \) represents the signal amplitude of the \( n \)th propagation path, \( f_{T,\text{max}} \) and \( f_{R,\text{max}} \) represent the maximum Doppler frequencies generated by the motion of the MS\(_T\) and MS\(_R\), respectively. In (2.28), the quantities \( \alpha_{T,n} \) and \( \alpha_{R,n} \) denote the angle of departure (AoD) and angle of arrival (AoA) of the \( n \)th path measured with respect to the MS\(_T\) and MS\(_R\) velocities vectors \( \vec{V}_T \) and \( \vec{V}_R \), respectively, as illustrated in Figure 2.7. In addition, \( \theta_n \) stands for the random phase uniformly distributed on \([-\pi, \pi]\). If \( N \to \infty \), i.e., sufficiently large \( N \) scatterers, then, we can invoke the central limit theorem [Papoulis 2002] to show that the process \( g_{si}(t) \) \((i = 1, 2)\) tends to a wide-sense stationary Gaussian process with zero-mean and variance \( \sigma_s^2 = 1/2 \sum_{n=1}^{N} c_n^2 \). Thus, the resulting fading amplitude \(|g_s(t)| = \sqrt{g_{s1}^2(t) + g_{s2}^2(t)}\) and the channel phase \(\tan^{-1}\left(\frac{g_{s2}(t)}{g_{s1}(t)}\right)\), at any given time instant, are described by the Rayleigh distributed PDF, as expressed in (2.8), and the uniform distribution, respectively. By assuming omnidirectional transmitter and receiver antennas, a 2-D isotropic scattering environment in the horizontal plane of the antennas, and a large number of scatterers (i.e., \( N \to \infty \)) Akki et al. [Akki 1986] derived the so-called double-Doppler ACF for the process \( g_{si}(t) \) \((i = 1, 2)\). This statistical quantity represents the
ACF of M2M Rayleigh fading channels, and is found to be given by [Akki 1986]

\[
\Gamma_{g_{si}g_{si}}(\tau) = \mathbb{E}\{g_{si}(t)^*g_{si}(t + \tau)\} = 1, 2
\]

\[
= \sigma^2_s J_0(2\pi f_{T,max} \tau) J_0(2\pi f_{R,max} \tau).
\]  

(2.29)

It should be noted that the above M2M ACF is a product of two Bessel functions rather than a single Bessel function. For the special case when \(f_{T,max} = 0\) (or \(f_{R,max} = 0\)), i.e., the case of the conventional cellular channel where the scattering and mobility are restricted to the mobile station end alone, (2.29) reduces to the classical ACF for Jakes’ model given by (2.18). Hence, the M2M channels appear as a set of generalized channels with cellular F2M channels being a special case. The behavior of the double-Doppler ACF for various combinations of the maximum Doppler frequencies \(f_{T,max}\) and \(f_{R,max}\) can be studied from Figure 2.8.

![Figure 2.8: The double-Doppler ACF of M2M fading channels for various combinations of the maximum Doppler frequencies \(f_{T,max}\) and \(f_{R,max}\).](image)

According to (2.15), an expression for the corresponding M2M Doppler PSD can be obtained by taking the Fourier transform of (2.29). Therefore, the M2M double-Doppler PSD of the process \(g_{si}(t)\) \((i = 1, 2)\) is obtained to be [Akki 1986]

\[
S_{g_{si}g_{si}}(f) = \frac{\sigma^2_s}{\frac{\pi^2}{f_{T,max} f_{R,max}}} K \left[ \left( \frac{(f_{T,max} + f_{R,max})^2 - f^2}{4 f_{T,max} f_{R,max}} \right)^{1/2} \right]
\]  

(2.30)
2.2. An overview of the M2M communication channels

where $K(\cdot)$ stands for the complete elliptic integral function of the first kind [Gradshteyn 1994]. In Figure 2.9, we depict the Doppler PSD encountered in the M2M fading channels. It can be seen from this figure that the behavior of the M2M double-Doppler PSD differs from the well known $U$-shape spectrum encountered in the cellular F2M channels, i.e., the Jakes’ spectrum.

On the other hand, the M2M spectrum $S_{g_{si}g_{si}}(f)$ has two peaks at $f = \pm (f_{T,\text{max}} - f_{R,\text{max}})$, and the bandwidth of the corresponding spectrum is given by $2(f_{T,\text{max}} + f_{R,\text{max}})$ [Akki 1986]. The well-known Jakes’ model, i.e., $f_{T,\text{max}} = 40$ Hz and $f_{R,\text{max}} = 0$ Hz gives the maxima spectrum width. However, this spectrum width decreases continually when the value of the ratio of maximum frequencies $f_{R,\text{max}}$ and $f_{T,\text{max}}$, i.e., $f_{R,\text{max}}/f_{T,\text{max}}$, increases.

Apart from the PDF and ACF statistical quantities, higher order statistics have been derived for the M2M single Rayleigh fading model. Namely, expressions for the LCR and ADF of the M2M single Rayleigh fading process $R_s(t) = |g_s(t)|$ are given by [Akki 1994b]

\[
N_{R_s}(r) = \frac{\sqrt{\pi}}{\sigma_s} \left( f_{T,\text{max}}^2 + f_{R,\text{max}}^2 \right)^{1/2} r \exp \left[ -\frac{r^2}{2\sigma_s^2} \right] \tag{2.31}
\]

and

\[
T_{R_s}(r) = \frac{\sigma_s}{\sqrt{\pi}} \left( f_{T,\text{max}}^2 + f_{R,\text{max}}^2 \right)^{1/2} r \left( \exp \left[ -\frac{r^2}{2\sigma_s^2} \right] - 1 \right), \tag{2.32}
\]
2.2. An overview of the M2M communication channels

Figure 2.10: The LCR of M2M single Rayleigh fading channels for various combinations of the maximum Doppler frequencies $f_{T, \text{max}}$ and $f_{R, \text{max}}$.

Figure 2.11: The ADF of M2M single Rayleigh fading channels for various combinations of the maximum Doppler frequencies $f_{T, \text{max}}$ and $f_{R, \text{max}}$. 
2.2. An overview of the M2M communication channels

respectively [Akki 1994b]. Again, by letting \( f_{T,max} = 0 \) (or \( f_{R,max} = 0 \)), i.e., the case of the conventional F2M cellular channel, (2.31) and (2.32) reduce, respectively, to the LCR and ADF of the Rayleigh fading channel [Jakes 1993]. It should also be noted that for the special case when we have the same speed for the MS\(_T\) and MS\(_R\), i.e., \( f_{T,max} = f_{R,max} \), the comparison of (2.22) and (2.31) leads to conclude that the LCR of the M2M single Rayleigh fading channel increases by a factor of \( \sqrt{2} \) over the case of the conventional F2M Rayleigh channel. In Figure 2.10, we show the LCR of M2M single Rayleigh fading channels for different combinations of the maximum Doppler frequencies \( f_{T,max} \) and \( f_{R,max} \). As it can be seen, the LCR increases with the increase of the MS\(_T\) and MS\(_R\) speed.

The ADF of the M2M single Rayleigh fading channels is shown, in Figure 2.11, for various combinations of the maximum Doppler frequencies \( f_{T,max} \) and \( f_{R,max} \). It should be noted from this figure that, with the increase of the maximum Doppler frequencies of the MS\(_T\) and MS\(_R\), i.e., \( f_{T,max} \) and \( f_{R,max} \), respectively, the underlying ADF decreases. Hence, the reduction in fade durations for M2M channels is due to the faster fading caused by the increased speed.

2.2.2.2 The double-ring model

The second M2M single Rayleigh model assumes the double-ring scattering environment as shown in Figure 2.12. In this model, we define a ring of uniformly spaced scatterers around both the MS\(_T\) and MS\(_R\), giving rise to isotropic local scattering [Patel 2005]. Hence, by assuming omnidirectional antennas at both the transmitter and receiver ends, the received baseband complex channel gain, for narrowband transmissions over frequency-flat fading conditions, can

![Double-ring model diagram](image-url)
be written as [Patel 2005]

\[ g_d(t) = \lim_{M,N\to\infty} \sum_{k=1}^{M} \sum_{n=1}^{N} c_{kn} \exp\left[j (2\pi \{ f_{T,\max} \cos(\alpha_{T,k}) + f_{R,\max} \cos(\alpha_{R,n}) \} t + \theta_{kn})\right] \] (2.33)

where the index \( k \) refers to the paths traveling from the MS\(_T\) to the \( M \) scatterers located on the MS\(_R\) end ring, while the index \( n \) refers to the paths traveling from the MS\(_R\) to the \( N \) scatterers located on the MS\(_T\) end ring. In (2.33), \( c_{kn} \) and \( \theta_{kn} \) denote the joint amplitude and joint phase caused by the interaction of the \( M \) transmitter and \( N \) receiver scatterers.

The above double-ring scattering model appears as a more generic M2M model than the last single-ring model [Patel 2005]. However, all its statistical quantities, e.g., the PDF, ACF, Doppler PSD, etc., have been demonstrated to be the same as that already derived for the single-ring scattering model. Therefore, when an isotropic scattering is assumed, the double-ring model corresponds to the M2M single Rayleigh model.

### 2.2.3 The double Rayleigh model

#### 2.2.3.1 The general double-ring model

The general double-ring model also known as the so-called cascaded or double Rayleigh model has been presented in [Kovacs 2002a, Kovacs 2002b]. This model assumes the double-ring scattering propagation environment as shown in Figure 2.12. However, in the corresponding propagation scenario, we assume that the distance between the MS\(_T\) and MS\(_R\) is considered to be large. In this case, the complex received channel gain signal can be described by the product of two complex Gaussian processes according to [Kovacs 2002a, Kovacs 2002b]

\[ g_D(t) = \lim_{M\to\infty} \sum_{k=1}^{M} A_k \exp\left[j (2\pi f_{T,\max} \cos(\alpha_{T,k}) t + \theta_{T,k})\right] \times \lim_{N\to\infty} \sum_{n=1}^{N} B_n \exp\left[j (2\pi f_{R,\max} \cos(\alpha_{R,n}) t + \theta_{R,n})\right] \] (2.34)

where \( A_k \), \( \theta_{T,k} \), \( B_n \), and \( \theta_{R,n} \) are independently distributed random amplitudes and phases due to local scatterers around the MS\(_T\) and MS\(_R\), respectively. The expression (2.34) can also be expressed as

\[ g_D(t) = g_{D_1}(t) \times g_{D_2}(t) \] (2.35)
where \( g_{D1}(t) \) and \( g_{D1}(t) \) are given by

\[
g_{D1}(t) = \lim_{M \to \infty} \sum_{k=1}^{M} A_k \exp \left[ j (2\pi f_{T,\text{max}} \cos(\alpha_{T,k}) t + \theta_{T,k}) \right]
\]

and

\[
g_{D2}(t) = \lim_{N \to \infty} \sum_{n=1}^{N} B_n \exp \left[ j (2\pi f_{R,\text{max}} \cos(\alpha_{R,n}) t + \theta_{R,n}) \right],
\]

respectively. It should be noted that (2.35) is more generic than (2.33) because the signal amplitudes corresponding to the propagation paths are considered as independent random variables and not as a product of independent uniformly distributed random variables. The fading process of the corresponding fading channel is given by \( R_D(t) = |g_D(t)| \). By introducing the quantities \( R_{D1}(t) = |g_{D1}(t)| \) and \( R_{D2}(t) = |g_{D2}(t)| \), the process \( R_D(t) \) can be written as

\[
R_D(t) = R_{D1}(t) R_{D2}(t).
\]

The process \( R_D(t) \) is generally known as the double Rayleigh fading amplitude which corresponds to that of the product of the two independent Rayleigh processes \( R_{D1}(t) \) and \( R_{D2}(t) \). An analytical expression for the PDF of the double Rayleigh process \( R_D(t) \), which is known as the double Rayleigh distribution, has been derived in [Kovacs 2002a, Kovacs 2002b] and has the form

\[
p_{R_D}(z) = \frac{z}{\sigma_{D1}^2 \sigma_{D2}^2} K_0 \left( \frac{z}{\sigma_{D1} \sigma_{D2}} \right)
\]

where \( K_0(\cdot) \) is the zeroth order modified Bessel function of the second kind [Gradshteyn 1994]. Also in (2.39), \( \sigma_{D1}^2 \) and \( \sigma_{D2}^2 \) stand for the mean powers of the two independent Rayleigh processes \( R_{D1}(t) \) and \( R_{D2}(t) \), respectively. In Figure 2.13, we show the behavior of the amplitude PDF, \( p_{R_D}(z) \), of the M2M double Rayleigh model for various combinations of the mean powers \( \sigma_{D1}^2 \) and \( \sigma_{D2}^2 \). It should be noted from this figure that for the same mean powers values \( \sigma_{D1}^2 \) and \( \sigma_{D2}^2 \), i.e., \( \sigma_{D1}^2 = \sigma_{D1}^2 \), the PDF maxima and the PDF spread increase with the decrease and the increase of the mean power values \( \sigma_{D1}^2 \) and \( \sigma_{D2}^2 \) (\( \sigma_{D1}^2 = \sigma_{D1}^2 \)), respectively. The ACF of the double Rayleigh model, i.e., the model described by the process \( g_D(t) \), is also given by the so-called double-Doppler ACF expressed by (2.29) [Akki 1986].
2.2. An overview of the M2M communication channels

2.2.3.2 The Amplify-and-forward relay model

As it has been mentioned previously, a new paradigm for a wireless communication called cooperative communication has recently gained a widespread attention. This mobile communication technique appears as a new form of spatial diversity, also known as the cooperation diversity [Sendonaris 2003a, Sendonaris 2003b, Laneman 2004, Laneman 2001, Patel 2006]. Many types of relaying schemes have been proposed in the literature. Namely, fixed, selection, and incremental relaying methods have been studied. Recently, the amplify-and-forward (AAF) relaying system, first introduced in [Laneman 2004] as a fixed relaying model, has gained widespread attention because of its simplicity [Patel 2006, Talha 2007, Salo 2006b]. In such a cooperative diversity system, the signal transmitted by the MS$_T$ is first amplified by a RS and, then, sent to the MS$_R$. Therefore, the overall fading channel model between the MS$_T$ and MS$_R$, via the RS, can be modeled as a serial concatenation of two single-hop channel models, where the first one describes the multipath propagation from the MS$_T$ to the RS, while the second one accounts for the propagation modeling over the link from the RS to the MS$_R$. This channel is generally known as the double fading channel, i.e., the overall channel gain is obtained as the product of the gains of the two single-hop channels. Figure 2.14 shows a two-hop AAF communication scenario illustrating the link between the MS$_T$ and MS$_R$ via the fixed RS. Given the importance of the AAF relay model, it is becoming necessary to analyze their statistical properties. Recently,
by assuming that both signals on the uplink and the downlink propagation paths from the MS$_T$ to the RS and from the MS$_R$ to the RS, respectively, are described by a Rayleigh multipath fading, then, the propagation in AAF wireless relay channels can be modeled by a M2M double Rayleigh fading model [Patel 2006]. Therefore, the amplitude distribution of the received signal in the wireless AAF relay networks disturbed by the Rayleigh multipath fading is given by the so-called double Rayleigh distribution expressed in (2.39).

2.3 Conclusion

This chapter sought to provide an overview of the main data base on multipath fading channels, which are needed and required in the achievement of this work. First, the Hoyt multipath fading channel has been reviewed. Next, a brief review on the M2M communication channels has been presented. It can be noted from the existent theoretical analysis of the typical M2M channel that the received signal statistics can follow a single or double Rayleigh distribution depending on the scattering environments. However, for both the single and double Rayleigh fading model, the total Doppler PSD of M2M channels is given by the convolution of the two individual spectrums generated by the motion of the mobile transmitter and receiver, i.e., the so-called double-Doppler PSD.

By considering the Hoyt fading channel as described above, we present, in the next chapter, a study on the performance analysis of FSK modulation with LDI detection over a such channel perturbed by AWGN.
The investigation of the error rate performance of the FSK modulation with an LDI detection has been widely studied for the case of the Rayleigh and Rice fading channels \cite{Tjhung 1990, Ng 1994, Korn 1990a, Korn 1990b}. In \cite{Tjhung 1990, Ng 1994}, closed-form expressions for the BEP performance have been reported for a transmission over Rayleigh fading and the validity of the results has been verified by using experimental data. For the Rice fading model, the error analysis of $M$-FSK transmission has been studied in \cite{Korn 1990a, Korn 1990b}. By considering the Hoyt fading channel, a closed-form expression for the BEP of FSK modulation with the classical limiter discriminator (LD) detection has been derived \cite{Zhaounia 2004}. In this study, a rectangular shape for the predetection filter has been used and no integrate and dump filter has been assumed (sampling discriminator).

In this chapter, we present a performance analysis of a narrowband digital FSK modulation scheme with an LDI detection over the Hoyt mobile fading channels. First, we derive a closed-form expression for the PDF of the phase angle between two Hoyt fading vectors. Then, an expression for the PDF of the phase noise due to additive Gaussian noise, is presented. The average number of FM clicks occurring at the output of a digital FM receiver is expressed, in Section 3.4. Based on the theory of the BEP performance and all the underlying investigated quantities, a formula for the BEP performance of FSK with LDI detection scheme operating over Hoyt mobile fading channels is obtained. Then, we present numerical examples to illustrate the analysis and study the influence of the fading severity as well as the FM system parameters on the obtained BEP performance. Finally, Section 3.7 concludes the chapter.

### 3.1 FSK system model

The binary FSK system with an LDI detection under study is depicted, as a block diagram, in Figure 3.1. The transmitted FSK signal $s_t(t)$ can be expressed, in the equivalent complex
3.1. FSK system model

Baseband representation, as

\[ s_t(t) = \exp[j\theta(t)] \] (3.1)

where the data phase \( \theta(t) \) is given by

\[ \theta(t) = \frac{\pi h}{T} \int_{-\infty}^{t} b(\tau)d\tau. \] (3.2)

In (3.2), \( b(t) \) stands for the binary data sequence of bit rate \( 1/T \) and \( h \) denotes the FSK modulation index. After a transmission over the Hoyt fading channel, the received signal \( s_r(t) \) can be expressed as

\[ s_r(t) = R(t) \exp[\theta(t) + \vartheta(t)] \] (3.3)

where \( R(t) \) represents the Hoyt fading process given by (2.3) and \( \vartheta(t) \) denotes the Hoyt channel phase expressed in (2.4). The pre-detection IF bandpass filter is considered to be of a Gaussian shape with an equivalent lowpass transfer function given by

\[ H(f) = \exp[-\pi f^2/2B^2] \] (3.4)

where \( B \) stands for the equivalent noise bandwidth. Now, by assuming a slowly varying Hoyt fading, the IF filter output signal \( e_0(t) \) can be written as [Tjhung 1990]

\[ e_0(t) = R_0 a(t) \exp[\phi(t) + \vartheta(t)] + n(t) \] (3.5)

Figure 3.1: Digital FSK system model with LDI detection.
3.1. FSK system model

where $a(t)$ and $\phi(t)$ are the filtered carrier amplitude and signal phase, respectively. In (3.5), $n(t)$ represents the additive Gaussian noise of an average power $\sigma_n^2 = N_0B$, and in which $N_0$ is the corresponding one-sided power spectral density. In addition, in (3.5), $R_0$ is a Hoyt random variable, for which the variance of the process $\mu_{1i}(t)$ ($i = 1, 2$) is reduced to $\sigma^2_{i0}$ due to the IF filtering. By considering the Jakes’ model for the Doppler PSD [Jakes 1993], the reduced variance $\sigma^2_{i0}$ ($i = 1, 2$) is found to be given by [Tjhung 1990]

$$\sigma^2_{i0} = \frac{\sigma^2_{i1}}{\pi} K_i$$

(3.6)

where the quantity $K_i$ has the form

$$K_i = \left[ \exp(-k_i) + 1 \right] \frac{\pi}{2} + \frac{2\pi}{k_i} \exp\left(-k_i/2\right) \left\{ I_0 \left( \frac{k_i}{2} \right) - \cosh \left( \frac{k_i}{2} \right) \right\}.$$  \hspace{1cm} (3.7)

In (3.7), the parameter $k_i = \pi(f_{\text{max}i})^2/B^2$ where, as it has been mentioned, $f_{\text{max}i}$ denotes the maximum Doppler frequency corresponding to the Gaussian process $\mu_{1i}(t)$. Here, it is worth mentioning that in the analysis we assume different Doppler frequencies $f_{\text{max}1}$ and $f_{\text{max}2}$ for the processes $\mu_{11}(t)$ and $\mu_{12}(t)$, respectively. Although, this assumption lacks a clear physical basis, it allows to increase the flexibility of the Hoyt fading model and enables a better fitting of measurement data [Youssef 2005b]. Next, following [Pawula 1981, Tjhung 1990] the output of the limiter circuit can be expressed as

$$e_1(t) = \exp[j\psi(t)]$$

(3.8)

where $\psi(t) = \phi(t) + \vartheta(t) + \eta(t)$ represents the overall phase, with $\eta(t)$ is the phase caused by the additive Gaussian noise. Then, the LD circuit outputs the derivative of the phase $\psi(t)$ with respect to the time, i.e., $\dot{\psi}(t) = d\psi(t)/dt$. Now, the integrate-and-dump filter with an integration time $T$ integrates $\dot{\psi}(t)$ producing, at a sampling time $t_0$, the following expression for the overall phase difference

$$\Delta \Psi = \Delta \phi + \Delta \eta + \Delta \vartheta + 2\pi N(t_0 - T, t_0)$$

(3.9)

where $\Delta \phi = \phi(t) - \phi(t - T)$ denotes the data phase component, $\Delta \eta = \eta(t) - \eta(t - T)$ represents the continuous phase noise due to the additive Gaussian noise, $\Delta \vartheta = \vartheta(t) - \vartheta(t - T)$ is the phase difference introduced by the Hoyt fading channel, and $2\pi N$ stands for the click noise component generated in the time interval $[t_0 - T, t_0]$. Therefore, to evaluate the performance of
the LDI detection scheme when digital FM signals are transmitted over Hoyt fading channels, we need to calculate the PDF of the overall phase difference $\Delta \psi$. To obtain an expression for this PDF, the PDF’s of the difference phases $\Delta \eta$, $\Delta \vartheta$, and $2\pi N$ are mostly required. In fact, by making the assumption that the random variables $\Delta \eta$, $\Delta \vartheta$, and $N$ are statistically independent, then, the bit error probability $P_E$ of FSK with LDI detection over Hoyt fading channels can be expressed as [Pawula 1981, Tjhung 1990]

$$P_E = \text{Prob}(\Delta \Omega > \Delta \phi) + \bar{N}$$  \hspace{1cm} (3.10)

where Prob(·) stands for probability, $\Delta \Omega = \Delta \eta + \Delta \vartheta$, $\bar{N}$ denotes the average number of clicks. Also in (3.10), $\text{Prob}(\Delta \Omega > \Delta \phi)$ represents the probability that the phase difference $\Delta \Omega$ exceeds some angle $\Delta \phi$. This probability can be written as [Tjhung 1990]

$$\text{Prob}(\Delta \Omega > \Delta \phi) = \int_{\Delta \phi-\pi}^{\Delta \phi+\pi} d\varphi_2 \int_{\Delta \phi-\varphi_2}^{\Delta \phi+\varphi_2} p_{\Delta \vartheta}(\varphi_2) p_{\Delta \eta}(\varphi_1) d\varphi_1$$  \hspace{1cm} (3.11)

where $p_{\Delta \vartheta}(\varphi)$ and $p_{\Delta \eta}(\varphi)$ are the PDF’s of the difference phases $\Delta \vartheta$ and $\Delta \eta$, respectively. In the following, we present the derivation of all these quantities.

### 3.2 PDF $p_{\Delta \vartheta}(\varphi)$ of the phase difference $\Delta \vartheta$ due to Hoyt fading

As it has been discussed previously in chapter 2, the Hoyt channel gain is modeled, in the equivalent complex baseband description, by the complex process $\mu(t)$ given by

$$\mu(t) = \mu_1(t) + j\mu_2(t)$$  \hspace{1cm} (3.12)

where $\mu_1(t)$ and $\mu_2(t)$ are uncorrelated zero-mean Gaussian processes with the variances $\sigma^2_{11}$ and $\sigma^2_{12}$, respectively. The starting point for the determination of $p_{\Delta \vartheta}(\varphi)$ is the joint PDF of the Gaussian processes $\mu_1 = \mu_1(t)$, $\mu_2 = \mu_2(t)$, $\mu_3 = \mu_1(t+\tau)$, and $\mu_4 = \mu_2(t+\tau)$, considered at the output of the IF filter. This joint PDF can easily be shown to be obtained according to [Voelcker 1960]

$$p_{\mu_1,\mu_2,\mu_3,\mu_4}(\nu_1,\nu_2,\nu_3,\nu_4) = p_{\mu_1}(\nu_1)p_{\mu_3/\mu_1}(\nu_3) \cdot p_{\mu_2}(\nu_2)p_{\mu_4/\mu_2}(\nu_4)$$  \hspace{1cm} (3.13)

where $p_{\mu_3/\mu_1}(\nu_3)$ and $p_{\mu_4/\mu_2}(\nu_4)$ denote the conditional PDF’s of the processes $\mu_3$ and $\mu_4$ given $\mu_1$ and $\mu_2$, respectively. Using [Voelcker 1960, eq. (2)], allows us to get the following expression
for the joint PDF $p_{\mu_1\mu_2\mu_3\mu_4}(\nu_1, \nu_2, \nu_3, \nu_4)$

$$p_{\mu_1\mu_2\mu_3\mu_4}(\nu_1, \nu_2, \nu_3, \nu_4) = A_1 \exp \left[ -A_2 \left\{ \sigma_1^2 \nu_1^2 + \sigma_2^2 \nu_2^2 + \sigma_3^2 \nu_3^2 + \sigma_4^2 \nu_4^2 - 2 \left( \sigma_0^2 \kappa \rho_{\nu_1} \nu_1 \nu_3 + \sigma_0^2 \lambda \rho_{\nu_2} \nu_2 \nu_4 \right) \right\} \right]$$  \hspace{1cm} (3.14)

where $\rho_{\nu_i} (i = 1, 2)$ is the normalized ACF of the process $\mu_{1i}(t)$ and the quantities $\lambda, \kappa, A_1,$ and $A_2$ are, respectively, given by

$$\lambda = 1 - \rho_{\nu_1}^2$$

$$\kappa = 1 - \rho_{\nu_2}^2$$

$$A_1 = \frac{1}{4\pi^2 \sigma_1^2 \sigma_2^2 \sqrt{\lambda \kappa}}$$

$$A_2 = \frac{1}{2\sigma_1^2 \sigma_2^2 \lambda \kappa}.$$  \hspace{1cm} (3.15)

Now, by making use of the variable transformation given by

$$\begin{align*}
\mu_1 &= R_1 \cos \vartheta_1 \\
\mu_2 &= R_1 \sin \vartheta_1
\end{align*}$$

$$\begin{align*}
\mu_3 &= R_2 \cos \vartheta_2 \\
\mu_4 &= R_2 \sin \vartheta_2
\end{align*}$$  \hspace{1cm} (3.16)

we obtain the following expression for the joint PDF $p_{R_1R_2\vartheta_1\vartheta_2}(z_1, z_2, \varphi_1, \varphi_2)$ of the variables $R_1, R_2, \vartheta_1,$ and $\vartheta_2$

$$p_{R_1R_2\vartheta_1\vartheta_2}(z_1, z_2, \varphi_1, \varphi_2) = A_1 z_1 z_2 \exp \left[ -A_2 \left\{ z_1^2 \left( \sigma_0^2 \kappa \cos^2 \varphi_1 + \sigma_0^2 \lambda \sin^2 \varphi_1 \right) + z_2^2 \left( \sigma_0^2 \kappa \cos^2 \varphi_2 + \sigma_0^2 \lambda \sin^2 \varphi_2 \right) - 2z_1 z_2 \left( \sigma_0^2 \kappa \rho_{\nu_1} \cos \varphi_1 \cos \varphi_2 + \sigma_0^2 \lambda \rho_{\nu_2} \sin \varphi_1 \sin \varphi_2 \right) \right\} \right]$$  \hspace{1cm} (3.17)

where $0 \leq z_1, z_2 < \infty$ and $-\pi \leq \varphi_1, \varphi_2 < \pi$. Therefore, the joint PDF $p_{\vartheta_1\vartheta_2}(\varphi_1, \varphi_2)$ of the phases $\vartheta_1$ and $\vartheta_2$ can be computed according to

$$p_{\vartheta_1\vartheta_2}(\varphi_1, \varphi_2) = \int_{0}^{\infty} \int_{0}^{\infty} p_{R_1R_2\vartheta_1\vartheta_2}(z_1, z_2, \varphi_1, \varphi_2) \, dz_1 \, dz_2.$$  \hspace{1cm} (3.18)

The integrals involved in (3.18) can be solved analytically by making use of the multiple integrals evaluation approach reported in [Rice 1944, Rice 1945]. Based on that approach, we start by
considering the integral $I$ given by

$$I = \int_0^\infty \int_0^\infty \exp \left[ -A_2 \sigma_{10}^2 E z_1^2 - A_2 \sigma_{10}^2 F z_2^2 + 2A_2 \sigma_{10}^2 D z_1 z_2 \right] dz_1 dz_2 \quad (3.19)$$

where the quantities $E$, $F$, and $D$ are expressed by

$$E = q_0^2 \kappa \cos^2 \phi_1 + \lambda \sin^2 \phi_1$$
$$F = q_0^2 \kappa \cos^2 \theta_2 + \lambda \sin^2 \phi_2$$
$$D = q_0^2 \kappa \rho_1 \cos \phi_2 \cos \phi_2 + \lambda \rho_2 \sin \phi_1 \sin \phi_2 \quad (3.20)$$

and in which, the quantity $q_0$ represents the reduced Hoyt fading parameter given by

$$q_0 = \frac{\sigma_{20}}{\sigma_{10}} = q \cdot \frac{K_2}{K_1}. \quad (3.21)$$

In (3.21), $q$ stands for the Hoyt fading parameter defined by (2.6). It should be mentioned that for the special case corresponding to $f_{\text{max}} = f_{\text{max}_2}$, we have $K_1 = K_2$ and therefore $q_0 = q$.

To solve the integrals in (3.19), we first let $x_1 = z_1 \sigma_{10} (A_2 E)^{1/2}$ and $x_2 = z_2 \sigma_{10} (A_2 F)^{1/2}$. This leads to

$$I = \frac{1}{A_2 \sigma_{10}^2 (EF)^{1/2}} \int_0^\infty \int_0^\infty \exp \left[ -x_1^2 - x_2^2 + 2gx_1 x_2 \right] dx_1 dx_2 \quad (3.22)$$

where the parameter $g = \frac{D}{(EF)^{1/2}}$. Then, we make the following linear change of variables

$$\begin{cases}
  x_1 = y_1 + g \left( 1 - g^2 \right)^{-1/2} y_2 \\
  x_2 = \left( 1 - g^2 \right)^{-1/2} y_2
\end{cases} \quad (3.23)$$

which results in

$$- x_1^2 - x_2^2 + 2gx_1 x_2 = -y_1^2 - y_2^2. \quad (3.24)$$

Therefore, by applying these transformations, the integral $I$ can be written as

$$I = \frac{(1 - g^2)^{-1/2}}{A_2 \sigma_{10}^2 (EF)^{1/2}} \int_0^\infty \int_{-g(1-g^2)^{-1/2}}^\infty \exp \left[ -y_1^2 - y_2^2 \right] dy_1. \quad (3.25)$$
Figure 3.2: The PDF $p_{\Delta \theta}(\varphi)$ for various values of the normalized ACF $\rho_{\tau_1}$.

Now, by making use of the transformation of the cartesian coordinates $(y_1, y_2)$ to polar coordinates, and performing the necessary algebraic manipulations, the integral $I$ is given by

$$I = \frac{1}{2A_2\sigma_1^2} \frac{1}{(EF)^{1/2}} \frac{1}{(1 - g^2)^{1/2}} \cot^{-1} \left( -\frac{g}{(1 - g^2)^{1/2}} \right). \quad (3.26)$$

Finally, differentiating (3.26), with respect to the variable $g$, results in the following expression for the joint PDF $p_{\vartheta_1, \vartheta_2}(\varphi_1, \varphi_2)$

$$p_{\vartheta_1, \vartheta_2}(\varphi_1, \varphi_2) = \frac{q_0^2 (\lambda \kappa)^{3/2}}{4\pi^2} \frac{1}{(EF - D^2)^{1/2}} \times \left[ 1 + \frac{D}{(EF - D^2)^{1/2}} \right] \left[ \frac{\pi}{2} + \tan^{-1} \left( \frac{D}{(EF - D^2)^{1/2}} \right) \right]. \quad (3.27)$$

Next, by noting that $\vartheta_2 - \vartheta_1 = \Delta \theta$, the desired PDF $p_{\Delta \theta}(\varphi)$ of the phase difference $\Delta \theta$, due to the Hoyt multipath fading, can be obtained from (3.27) according to

$$p_{\Delta \theta}(\varphi) = \int_{-\pi}^{\pi} p_{\vartheta_1, \vartheta_2}(\varphi_1, \varphi_1 + \varphi) d\varphi_1. \quad (3.28)$$

Unfortunately, the integral involved in (3.28) can be evaluated only by using numerical techniques. It should be mentioned that for $f_{\text{max}} = f_{\text{max}}$ and $q = 1$, i.e., the case of the Rayleigh fading channel, the calculation of the above integral using (3.15), (3.20), (3.27), and
3.3 PDF $p_{\Delta \eta}(\varphi)$ of the phase difference $\Delta \eta$ due to the additive Gaussian noise

For the derivation of the PDF $p_{\Delta \eta}(\varphi)$ of the phase difference $\Delta \eta$ caused by the additive Gaussian noise, we use the result given by [Pawula 1981, Tjhung 1990]

$$p_{\Delta \eta|R_0}(\varphi) = \frac{\exp(-U)}{2\pi} \left\{ \cosh V + \frac{1}{2} \int_0^\pi d\alpha \left[ (U \sin \alpha + \sqrt{U^2 - V^2} \cos \varphi) \cdot \cosh (V \cos \alpha) \right] \times \exp \left( \sqrt{U^2 - V^2} \sin \alpha \cos \varphi \right) \right\}$$

3.3 PDF $p_{\Delta \phi}(\varphi)$ of the phase difference $\Delta \phi$ due to the additive Gaussian noise

As expected, this PDF corresponds to that of the phase difference due to the Rayleigh fading [Tjhung 1990, eq. (A4)]. The behavior of $p_{\Delta \phi}(\varphi)$, for $f_{\text{max}1} = f_{\text{max}2}$, $q = 0.447$, and various values of the normalized ACF $\rho_{\text{r1}}$, can be studied from Figure 3.2. In Figure 3.3, we show the effect of the Hoyt fading parameter $q$ on the PDF $p_{\Delta \phi}(\varphi)$ for $f_{\text{max}1} = f_{\text{max}2}$ and $\rho_{\text{r1}} = 0.7$. As can be seen, the lowest PDF maxima and the largest PDF spread correspond to that of the PDF of the phase difference $\Delta \phi$ between two Rayleigh fading processes, i.e., $q = 1$.
which represents the conditional PDF of the phase difference $\Delta \eta$ given the reduced Hoyt random variable $R_0$. In (3.30), the quantities $U$ and $V$ are obtained to be

$$U = \frac{1}{2} [\gamma(t_0) + \gamma(t_0 - T)] = \frac{R_0^2}{2\sigma_n^2} C_1$$

$$V = \frac{1}{2} [\gamma(t_0) - \gamma(t_0 - T)] = \frac{R_0^2}{2\sigma_n^2} C_2.$$

(3.31)

where the parameters $C_1$ and $C_2$ are functions of the specific bit patterns to be considered in the evaluation of the BEP performance. Expressions for these parameters can be found in the Appendix A. Also, in (3.31) the quantity $\gamma(t)$ is the time varying SNR defined by

$$\gamma(t) = \frac{R_0^2}{2} \frac{a^2(t)}{\sigma_n^2} = \frac{E_b}{N_0 BT} \frac{1}{(q^2 + 1) \sigma_n^2}$$

(3.32)

where $E_b = \frac{(q^2 + 1) \sigma_n^2}{2} T$ is the average received signal-energy-per-bit at the input of the IF filter. Then, the desired quantity for $p_{\Delta \eta}(\varphi)$ can be obtained by averaging $p_{\Delta \eta|R_0}(\varphi)$ over the reduced Hoyt random variable $R_0$ according to

$$p_{\Delta \eta}(\varphi) = \int_0^\infty p_{\Delta \eta|R_0}(\varphi)p_{R_0}(z)dz.$$

(3.33)

Using [Gradshteyn 1994, Eqs. 6.611(1), 6.623(2), and 8.406(3)], the PDF $p_{\Delta \eta}(\varphi)$ can be written as

$$p_{\Delta \eta}(\varphi) = d \left( \frac{1}{\sqrt{[(C_1 - C_2) + e]^2 - l^2}} + \frac{1}{\sqrt{[(C_1 + C_2) + e]^2 - l^2}} \right)$$

$$+ \frac{d}{2} \int_0^\pi d\alpha \left\{ (C_1 \sin \alpha + C_3 \cos \varphi) \right. \times$$

$$\left[ \frac{e - (C_3 \sin \alpha \cos \varphi + C_2 \cos \alpha - C_1)}{\left\{ e - (C_3 \sin \alpha \cos \varphi + C_2 \cos \alpha - C_1)^2 - l^2 \right\}^{3/2}} \right.$$

$$\left. + \frac{e - (C_3 \sin \alpha \cos \varphi - C_2 \cos \alpha - C_1)}{\left\{ e - (C_3 \sin \alpha \cos \varphi - C_2 \cos \alpha - C_1)^2 - l^2 \right\}^{3/2}} \right\} \right) \}$$

(3.34)
where the quantities $C_3$, $d$, $e$, and $l$ are given by

\begin{align*}
C_3 &= \sqrt{C_1^2 - C_2^2} \\
d &= \frac{1 + q_0^2}{8\pi \sqrt{\sigma}} \\
e &= \frac{(1 + q_0^2)^2}{4q_0^2} r \\
l &= \frac{1 - q_0^4}{4q_0^2} r.
\end{align*}

In (3.35), \( r = \frac{2\sigma^2}{(1+q_0^2)^2 t_0} \). Unfortunately, the integral in (3.34) is difficult to handle, and it can be evaluated only by using numerical techniques.

### 3.4 Average number of FM clicks $\overline{N}$

Based on the assumption that the FM clicks, at the output of the discriminator circuit, can be described statistically by a Poisson distributed discrete random variable [Rice 1963], the average number of clicks $\overline{N}$ occurring during the time interval $[t_0 - T, t_0]$, corresponding to the bit duration $T$, can be derived according to the following relation [Pawula 1981, Tjhung 1990]

\[
\overline{N} = \frac{1}{2\pi} \int_0^t \int_{t_0-T}^{t} \hat{\phi}(\tau) \exp\left(-\frac{z^2 a^2(\tau)}{2\sigma^2_n}\right) p_{R_0}(z) d\tau dz.
\]

(3.36)

After some algebraic manipulations, we obtain the following quantity for the desired $\overline{N}$

\[
\overline{N} = 2d \int_{t_0-T}^{t_0} \frac{\hat{\phi}(\tau)}{\sqrt{[(a^2(\tau) + e)^2 - l^2]}} d\tau.
\]

(3.37)

For the special case corresponding to $f_{\text{max}1} = f_{\text{max}2}$ and $q = 1$, (3.37) simplifies to the result given by

\[
\overline{N} = \frac{1}{2\pi \sigma_{\text{R0}}^2} \int_{t_0-T}^{t_0} \frac{\hat{\phi}(\tau)}{a^2(\tau) + \sigma^2_n / \sigma_{\text{R0}}^2} d\tau.
\]

(3.38)

As expected, this quantity corresponds perfectly to the average number of FM clicks, taking into account the Rayleigh multipath fading effects, that has been derived in [Tjhung 1990].

To summarize, the quantity given by (3.10), i.e., the BEP performance of the FSK with an LDI detection over Hoyt fading channels, can be evaluated using (3.11), (3.28), (3.34), and (3.37).
3.5 Bit error rate probability

According to [Pawula 1981, Tjhung 1990], for $BT \geq 1$, the distortion caused by the intersymbol interference (ISI), which are due to the bandwidth limitation of the pre-detection bandpass IF filter, extends only to one bit either side of the bit under detection. Thus, if we consider that a “1” bit is sent, the four bit patterns of “010”, “111”, “011”, and “110” will intervene in the calculation of the desired average BEP. By symmetry, “011” and “110” can be considered to have the same ISI effects upon the central bit. Therefore, only the three bit patterns given by “010”, “111”, and “011”, will be considered in the calculation of the desired BEP performance. For these three bit patterns, (3.10) can be approximated by [Tjhung 1990]

$$P_E \cong P_{e,1} + P_{e,2}$$

(3.39)

where

$$P_{e,1} = \frac{1}{4} \left[ \text{Prob}(\Delta \Omega > \Delta \phi | 111) + \text{Prob}(\Delta \Omega > \Delta \phi | 010) + 2\text{Prob}(\Delta \Omega > \Delta \phi | 011) \right]$$

(3.40)

and

$$P_{e,2} = \frac{1}{4} \left[ \overline{N}_{111} + \overline{N}_{010} + 2\overline{N}_{011} \right].$$

(3.41)

In (3.40), $\text{Prob}(\Delta \Omega > \Delta \phi | ...)$ represents the conditional error probability given the bit pattern “...” when the phase difference $\Delta \Omega$ exceeds some angle $\Delta \phi$, i.e., $\Delta \Omega > \Delta \phi$. Similarly, in (3.41), $\overline{N}_{...}$ denotes the conditional average number of FM clicks given the bit pattern “...”. From next obtained results, we can verify that the quantities $P_{e,1}$ and $P_{e,2}$ depend on $\dot{\phi}(t)$, $\Delta \phi$, $a^2(t)$, $C_1$, and $C_2$. The evaluation of these quantities must be performed for the three bits patterns given by “111”, “010”, and “011”. By considering that the sampling time is $t_0 = 0$, and according to [Tjhung 1990], all quantities needed for the calculation of the average BEP for the above bit patterns are shown in the Appendix A.

3.6 Results verification

In this section, we present computed numerical results for the average number of FM clicks $\overline{N}$ and the average BEP performance as a function of $E_b/N_0$ which is related to the parameter $r$, given in (3.35), according to

$$E_b/N_0 = \frac{\pi (1 + q^2)}{K_1 (1 + q_0^2)} \frac{BT}{r}.$$

(3.42)
3.6. Results verification

Figure 3.4 shows the average number of FM clicks $\overline{N}$, as a function of $E_b/N_0$, for $f_{\text{max}1} = f_{\text{max}2}$, $q = 0.447$, $h = 0.7$, the bit pattern “010”, and several values of the maximum Doppler frequency $f_{\text{max}1}$. As expected, $\overline{N}$ decreases as $E_b/N_0$ increases while it increases with increasing values of $f_{\text{max}1}$. In Figure 3.5, we study the effect of the Hoyt fading parameter $q$ on the average number of FM clicks $\overline{N}$ corresponding to the bit pattern “010” when $f_{\text{max}1} = f_{\text{max}2} = 20$ Hz, $BT = 1$, $h = 0.7$, and several values of the Hoyt fading parameter $q$. As it can be seen, the click noise decreases with the increase of the Hoyt fading parameter. The smallest FM click noise is obtained for $q = 1$, i.e., the case of the Rayleigh fading channel. For the computation of the BEP, we need to evaluate the normalized ACF $\rho_{\tau_i}$ ($i = 1, 2$), at $\tau = T$, i.e.,

$$\rho_{\tau_i} = \frac{1}{\sigma_{\tau_i}^2} \int_{-f_{\text{max}i}}^{f_{\text{max}i}} \frac{\sigma_i^2 |H(f)|^2}{\pi \sqrt{f_{\text{max}i}^2 - f^2}} \exp(j2\pi fT) \, df. \tag{3.43}$$

Using (3.6), (3.7), and [Tjhung 1990, eq. (A8)], (3.43) can be computed numerically according to

$$\rho_{\tau_i} = \frac{2}{K_i} \left[ \int_0^{\pi/2} \left\{ \exp \left( -p_i \sin^2 z \right) \cos (j_i \sin z) \right\} \, dz \right] \tag{3.44}$$
3.6. Results verification

Figure 3.5: The average number of FM clicks $\overline{N}$ for various values of the fading parameter $q$.

Figure 3.6: The BEP $P_E$ for various values of the Hoyt fading parameter $q$. 

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3.7 Conclusion

In this chapter, a performance analysis for narrowband FM systems with an LDI detection has been considered, in the case of the Hoyt fading channels. The PDF of the phase difference between two Hoyt vectors, and that induced by the additive noise, as well as the average number of FM clicks have been first derived. Based on all these quantities, an expression for the corresponding BEP performance has been deduced. Then, numerical results have been given, and by which the effects of the fading severity as well as the system parameters on the average number of FM clicks and the BEP performance have been examined. However, the methodology employed here consists of the determination, separately, of the PDF of the phase difference introduced by the receiver noise and that due to the Hoyt fading channel. Then,

where $p_i = \pi \left( f_{\text{max},i} T \right)^2 / (BT)^2$, and $j_i = 2\pi f_{\text{max},i} T$. In Figure 3.6, the average BEP is plotted versus $E_b/N_0$ for $h = 0.7$, $BT = 1$, $f_{\text{max},1}T = f_{\text{max},2}T = 0.004$, and several values of the fading parameter $q$. As can be seen, the error performance improves if $q$ increases. The best performance is obtained when $q = 1$, i.e., the case of the Rayleigh fading model. Figure 3.7 represents the average BEP $P_E$, as a function of $E_b/N_0$, for $q = 0.447$, $h = 0.7$, $f_{\text{max},1}T = f_{\text{max},2}T = 0.004$, and $BT = 1, 2, \text{ and } 3$. It can be noted from this figure that the performance improves with decreasing the values of the product $BT$.

3.7 Conclusion

Figure 3.7: The BEP $P_E$ for various values of $BT$. 

where $p_i = \pi \left( f_{\text{max},i} T \right)^2 / (BT)^2$, and $j_i = 2\pi f_{\text{max},i} T$. In Figure 3.6, the average BEP is plotted versus $E_b/N_0$ for $h = 0.7$, $BT = 1$, $f_{\text{max},1}T = f_{\text{max},2}T = 0.004$, and several values of the fading parameter $q$. As can be seen, the error performance improves if $q$ increases. The best performance is obtained when $q = 1$, i.e., the case of the Rayleigh fading model. Figure 3.7 represents the average BEP $P_E$, as a function of $E_b/N_0$, for $q = 0.447$, $h = 0.7$, $f_{\text{max},1}T = f_{\text{max},2}T = 0.004$, and $BT = 1, 2, \text{ and } 3$. It can be noted from this figure that the performance improves with decreasing the values of the product $BT$. 

3.7 Conclusion

In this chapter, a performance analysis for narrowband FM systems with an LDI detection has been considered, in the case of the Hoyt fading channels. The PDF of the phase difference between two Hoyt vectors, and that induced by the additive noise, as well as the average number of FM clicks have been first derived. Based on all these quantities, an expression for the corresponding BEP performance has been deduced. Then, numerical results have been given, and by which the effects of the fading severity as well as the system parameters on the average number of FM clicks and the BEP performance have been examined. However, the methodology employed here consists of the determination, separately, of the PDF of the phase difference introduced by the receiver noise and that due to the Hoyt fading channel. Then,
3.7. Conclusion

the resultant phase PDF, needed for the evaluation of the BEP expression, is calculated by a convolution operation, which demands tedious numerical integrations. In addition, the obtained BEP results are valid only for small values of the SNR ratio and is restricted only for the Doppler PSD of Jakes’ model.

To avoid the drawback of the method mentioned above, we will approach the problem of the BEP performance of FSK modulation with the LDI detection over the Hoyt fading channels from a different point of view. We will derive a closed-form expression for the PDF of the overall phase difference of a modulated Hoyt faded signal corrupted by additive Gaussian noise. The presentation of much more details on this new method will be the subject of the next chapter.
PDF of the phase difference between two Hoyt processes perturbed by Gaussian noise

For various digital modulation schemes, especially the DPSK and FSK with LDI and differential detection schemes, the decision variable needed for the demodulator is the differential phase, over a symbol period, of the received faded signal perturbed by the Gaussian noise [Pawula 1981, Xiong 2006]. For error rate calculations, an expression for the PDF of such a phase difference is vital information. Several previous works have considered the distribution of the phase angle between two faded vectors perturbed by Gaussian noise. The earliest published work has been presented in [Voelcker 1960], in which Voelcker derived the PDF of the phase difference between two Rayleigh fading processes. This PDF has, then, been applied in the investigation of the error rate performance of the PSK modulation over Rayleigh fading channels perturbed by the Gaussian noise. Apart from the PDF distribution, the CDF of the corresponding phase difference variable, can also be used in the calculation of the BEP performance of digital wireless communications over fading channels. In this framework, an expression for the CDF of the phase difference between two non-faded signals, each perturbed by the additive Gaussian noise has been derived in [Pawula 1982]. This CDF has been widely used in the error rate analysis of M-ary DPSK (MDPSK) modulation schemes, as it exemplified in [Mason 1987]. Then, the analysis on the CDF has been extended to the case of Rayleigh [Adachi 1992] and Nakagami faded signals [Fedele 1995]. Recently, Chai [Chai 1999] derived new expressions for the CDF of the phase difference between two signals perturbed by additive Gaussian noise, for mobile radio communications characterized by the so-called Rice-lognormal channels, i.e., Rician fading and lognormal shadowing, and Nakagami-lognormal channels, i.e., Nakagami fading and lognormal shadowing. From the literature above, only the PDF and CDF of the phase difference between two faded signals for the classical multipath fading models have been considered. To the best of our knowledge, for the case of the Hoyt fading channels, there exist no results on the distribution
4.1 PDF derivation

of the phase angle between two modulated Hoyt faded signals contaminated by Gaussian noise.
The aim of this study is to contribute to this topic.

Namely, the derivation of a closed-form expression for the PDF of the phase difference between two modulated Hoyt faded signals perturbed by correlated Gaussian noise is presented, in this chapter. The obtained PDF formula is verified to reduce to known results corresponding to the Rayleigh fading as a special case of the Hoyt fading model. Additionally, this PDF takes into account the Doppler spread effects and is valid for any Doppler PSD. For a further verification, the newly derived PDF is simulated using the deterministic Rice’s sum-of-sinusoids concept [Rice 1944, Rice 1945]. A comparison of the theoretical results with corresponding simulation ones, obtained for a M2M single Hoyt fading channel, are found to be in close agreements.

The remainder of the chapter is organized as follows. Section 4.1 is devoted to the derivation of the closed-form expression for the PDF of the differential phase between two Hoyt faded signals perturbed by correlated Gaussian noise. In Section 4.2, the validity of the obtained PDF formula is checked by means of the computer simulations. Finally, the conclusion is drawn in Section 4.3.

4.1 PDF derivation

In this section, we focus on the derivation of the PDF of the phase difference between two Hoyt faded sinusoid signals perturbed by correlated Gaussian noise. As a starting point, we show in Figure 4.1, as a block diagram, the digital transmitter and receiver under study. After phase or

![Figure 4.1: Block diagram of digital communication systems over Hoyt fading channels.](image)

frequency modulation of the input signal, the modulator outputs the signal $s(t)$ given by

$$s(t) = \exp[j\phi(t)]$$  \hspace{1cm} (4.1)
4.1. PDF derivation

where $\phi(t)$ denotes the data phase after an angular modulation. The channel between the transmitter and receiver is modeled as a frequency-flat Hoyt multipath fading. After the propagation through the underlying fading channel, the arrival signal at the receiver is combined with a correlated Gaussian noise $n(t)$. This results in the following complex signal

$$e(t) = \mu_1(t) \exp [j\phi(t)] + n(t) \tag{4.2}$$

where $\mu_1(t)$ represents the complex Hoyt channel gain process given by (2.1) and $n(t) = n_1(t) + jn_2(t)$ with $n_1(t)$ and $n_2(t)$ are the in-phase and quadrature zero-mean Gaussian noise components, respectively. These components have a common variance and an ACF denoted here by $\sigma_n^2$ and $\Gamma_{nn}(\tau)$, respectively. Due to the Hoyt fading and Gaussian noise effects, the overall phase of the signal $e(t)$ can be expressed as

$$\psi(t) = \phi(t) + \Omega(t) \tag{4.3}$$

where $\Omega(t)$ stands for the phase caused by the fading plus additive noise. To derive an expression for the PDF of the phase difference $\Delta\Omega = \Omega(t) - \Omega(t - \tau)$ between two Hoyt faded signals perturbed by the additive Gaussian noise, we consider the signal $e(t)$ at two time instants.

Figure 4.2: Phasor diagram of a modulated Hoyt faded signal contaminated by Gaussian noise.
4.1. PDF derivation

t_1 = t - \tau \text{ and } t_2 = t, \text{ i.e., } e(t_1) \text{ and } e(t_2). \text{ By assuming a coordinate system that rotates with}
a phase angle \phi_1 = \phi(t_1) \text{ as shown in Figure 4.2, we may write for the processes } e(t_1) \text{ and } e(t_2)\nthe following quantities given by

\begin{align*}
  z_1 &= (\mu_{11} + \zeta_1) + j(\mu_{12} + \xi_1) \\
     &= x_1 + jy_1 \\
     &= R_1 \exp[j\psi_1] \quad (4.4)
\end{align*}

and

\begin{align*}
  z_2 &= (\mu_{21} \cos(\Delta \phi) - \mu_{22} \sin(\Delta \phi) + \zeta_2) + j(\mu_{21} \sin(\Delta \phi) + \mu_{22} \cos(\Delta \phi) + \xi_2) \\
     &= x_2 + jy_2 \\
     &= R_2 \exp[j\psi_2] \quad (4.5)
\end{align*}

respectively. In the above expressions, \Delta \phi = \phi(t) - \phi(t - \tau) \text{ is the data phase difference,}
\mu_{1i} = \mu_{1i}(t_1) \text{ and } \mu_{2i} = \mu_{1i}(t_2) \text{ (} i = 1, 2). \text{ Also in (4.4) and (4.5), the processes } \zeta_i \text{ and } \xi_i
\text{ (} i = 1, 2) \text{ denote the noise components defined relatively to the coordinate system that rotates}
with } \phi_1, \text{ and are, respectively, related to the noise components } n_{1i} = n_i(t_1), \text{ and } n_{2i} = n_i(t_2)
\text{ (} i = 1, 2) \text{ according to}

\begin{align*}
  \zeta_i &= n_{1i} \cos(\phi_1) + n_{i2} \sin(\phi_1) \quad (4.6)
\end{align*}

and

\begin{align*}
  \xi_i &= n_{i2} \cos(\phi_1) - n_{i1} \sin(\phi_1). \quad (4.7)
\end{align*}

It can easily be shown that these components are zero-mean independent Gaussian random
variables having the same variance \sigma_n^2 \text{ and the ACF } \Gamma_{nn}(\tau) \text{ corresponding to that of the noise}
n_{ij} \text{ (} i, j = 1, 2). \text{ Thus, it may be concluded that, in this new coordinate system, the statistical}
properties of the resulting noise components } \xi_i \text{ and } \zeta_i \text{ remain unchanged with respect to those}
of } n_{ij} \text{ (} i, j = 1, 2). \text{ For the sake of convenience, we denote the quantities } \mu_{11} + \zeta_1 \text{ and } \mu_{12} + \xi_1,
\text{ respectively, by } x_1 \text{ and } y_1, \text{ for the quadrature components of the signal } z_1. \text{ Similarly, for } z_2,
\text{ we denote the quantities } \mu_{21} \cos(\Delta \phi) - \mu_{22} \sin(\Delta \phi) + \zeta_2 \text{ and } \mu_{21} \sin(\Delta \phi) + \mu_{22} \cos(\Delta \phi) + \xi_2 \text{ by}
x_2 \text{ and } y_2, \text{ respectively. From expressions (4.4) and (4.5), we can note that } x_i \text{ and } y_i \text{ (} i = 1, 2)\text{ are combinations of Gaussian processes. Hence, for a given information symbol } \Delta \phi, \text{ they are}
Gaussian distributed as well. Since we assume a symmetrical Doppler PSD for the Hoyt fading and uncorrelated additive noise, we have

\[ E \{ \mu_{11} \mu_{12} \} = E \{ \mu_{21} \mu_{22} \} = E \{ \zeta_1 \xi_1 \} = E \{ \zeta_2 \xi_2 \} = 0. \quad (4.8) \]

Furthermore, assuming that the signal component is independent of the noise component allows us to obtain the following expressions for the cross-correlation quantities of the variables \( x_1, y_1, x_2, \) and \( y_2 \)

\[
\begin{align*}
a_{12} &= E \{ x_1 y_1 \} = 0 \\
a_{14} &= E \{ x_1 y_2 \} = \Gamma_{\mu_{11} \mu_{11}}(\tau) \sin(\Delta \phi) \\
a_{32} &= E \{ x_2 y_1 \} = -\Gamma_{\mu_{12} \mu_{12}}(\tau) \sin(\Delta \phi) \\
a_{34} &= E \{ x_2 y_2 \} = \sigma_{11}^2 (1 - q^2) \cos(\Delta \phi) \sin(\Delta \phi) \\
a_{13} &= E \{ x_1 x_2 \} = \Gamma_{\mu_{11} \mu_{11}}(\tau) \cos(\Delta \phi) + \Gamma_{nn}(\tau) \\
a_{24} &= E \{ y_1 y_2 \} = \Gamma_{\mu_{12} \mu_{12}}(\tau) \cos(\Delta \phi) + \Gamma_{nn}(\tau) \end{align*} \quad (4.9)
\]

where \( \Gamma_{\mu_{11} \mu_{11}}(\tau) \) \( (i = 1, 2) \) represents the ACF of the process \( \mu_{11}(t) \) \( (i = 1, 2) \). Concerning the mean power of the underlying Gaussian processes, it is found to be given by

\[
\begin{align*}
\sigma_{x_1}^2 &= E \{ x_1^2 \} = \sigma_n^2 (\Lambda + 1) \\
\sigma_{y_1}^2 &= E \{ y_1^2 \} = \sigma_n^2 (q^2 \Lambda + 1) \\
\sigma_{x_2}^2 &= E \{ x_2^2 \} = \sigma_n^2 (\Lambda \{ \cos^2(\Delta \phi) + q^2 \sin^2(\Delta \phi) \} + 1) \\
\sigma_{y_2}^2 &= E \{ y_2^2 \} = \sigma_n^2 (\Lambda \{ \sin^2(\Delta \phi) + q^2 \cos^2(\Delta \phi) \} + 1) \end{align*} \quad (4.10)
\]

where the parameter \( \Lambda = \sigma_{11}^2 / \sigma_n^2 \). Finally, in addition to the quadrature signal based description presented above, we introduce the quantities \( R_i = \sqrt{x_i^2 + y_i^2} \) and \( \psi_i = \tan^{-1}(y_i/x_i)(i = 1, 2) \), to denote, respectively, the overall envelope and phase of the modulated Hoyt faded signal contaminated by correlated Gaussian noise. As it has previously been mentioned, we are concerned mostly with the derivation of the PDF \( p_{\Delta \Omega}(\varphi) \) of the phase difference \( \Delta \Omega \) due to the Hoyt fading plus an additive noise. A starting point for the derivation of this PDF is the determination of the joint PDF \( p_{x_1 y_1 x_2 y_2}(\nu_1, \nu_2, \nu_3, \nu_4) \) of the Gaussian random variables \( x_1, y_1, x_2, \) and \( y_2 \). Based on the multivariate Gaussian distribution, the joint PDF \( p_{x_1 y_1 x_2 y_2}(\nu_1, \nu_2, \nu_3, \nu_4) \) can be
written as

$$p_{x_1y_1x_2y_2} (\nu_1, \nu_2, \nu_3, \nu_4) = \frac{1}{(2\pi)^2 \sqrt{\det \Sigma}} \exp \left[ (\nu_1, \nu_2, \nu_3, \nu_4)^H \Sigma^{-1} (\nu_1, \nu_2, \nu_3, \nu_4) \right]$$

(4.11)

where \(\Sigma\) represents the covariance matrix of the vector process \((x_1, y_1, x_2, y_2)^H\) and in which \(H\) stands for the transpose operator. Also, in (4.11) \(\Sigma^{-1}\) is the inverse matrix of the covariance matrix \(\Sigma\) and \(\det \Sigma\) denotes the determinant of \(\Sigma\). Using (4.9) and (4.10), the covariance matrix \(\Sigma\) can be expressed as

$$\Sigma = \begin{pmatrix} \sigma^2_{x_1} & 0 & a_{13} & a_{14} \\ 0 & \sigma^2_{y_1} & a_{23} & a_{24} \\ a_{13} & a_{23} & \sigma^2_{x_2} & a_{34} \\ a_{14} & a_{24} & a_{34} & \sigma^2_{y_2} \end{pmatrix}. \quad (4.12)$$

After some algebraic manipulations, the determinant of this matrix, i.e., \(\det \Sigma\), is found to be given by

$$\det \Sigma = \sigma^2_{x_1} \sigma^2_{y_1} \sigma^2_{x_2} \sigma^2_{y_2} \cdot K \quad (4.13)$$

where the quantity \(K\) is expressed as a function of the correlation quantities of the variables \(x_1, y_1, x_2, \) and \(y_2\) and is shown in the Appendix B. Then, a calculation of the inverse matrix of \(\Sigma\) gives

$$\Sigma^{-1} = \frac{1}{\det \Sigma} \begin{pmatrix} \chi_{11} & -\chi_{12} & \chi_{13} & -\chi_{14} \\ -\chi_{12} & \chi_{22} & -\chi_{23} & \chi_{24} \\ \chi_{13} & -\chi_{23} & \chi_{33} & -\chi_{34} \\ -\chi_{14} & \chi_{13} & -\chi_{34} & \chi_{44} \end{pmatrix} \quad (4.14)$$

where the parameters \(\chi_{ij} \) \((i, j = 1, \ldots, 4)\) are listed in the Appendix B. Therefore, substituting (4.13) and (4.14) in (4.11) and doing some algebraic manipulations yields the following expression for the joint PDF \(p_{x_1y_1x_2y_2} (\nu_1, \nu_2, \nu_3, \nu_4)\) of the variables \(x_1, y_1, x_2, \) and \(y_2\)

$$p_{x_1y_1x_2y_2} (\nu_1, \nu_2, \nu_3, \nu_4) = B_1 \exp \left[ -B_2 \left[ (\chi_{11} \nu_1^2 + \chi_{22} \nu_2^2 + 2\chi_{12} \nu_1 \nu_2) \right. \right.$$

$$+ \left. (\chi_{33} \nu_3^2 + \chi_{44} \nu_4^2 + 2\chi_{34} \nu_3 \nu_4) \right]$$

$$- 2 (\nu_1 \{\chi_{13} \nu_3 + \chi_{14} \nu_4\} + \nu_2 \{\chi_{23} \nu_3 + \chi_{24} \nu_4\}) \right] \right] \quad (4.15)$$
where the quantities $B_1$ and $B_2$ are given by

$$B_1 = \frac{1}{4\pi^2 \sigma_x \sigma_y \sigma_1 \sigma_2 \sigma_3 K^{1/2}}, \quad B_2 = \frac{1}{2K}. \quad (4.16)$$

Now, the transformation of the Cartesian coordinates $(x, y)$, to polar coordinates $(R, \psi)$, results in the following expression for the joint PDF $p_{R_1, R_2 \psi_1 \psi_2}(z_1, z_2, \varphi_1, \varphi_2)$ of the random variables $R_1$, $R_2$, $\psi_1$, and $\psi_2$, according to

$$p_{R_1, R_2 \psi_1 \psi_2}(z_1, z_2, \varphi_1, \varphi_2) = B_1 z_1 z_2 \exp \left[ -B_2 \left( \left( \chi_{11} \cos^2 \varphi_1 + \chi_{22} \sin^2 \varphi_1 + 2\chi_{12} \cos \varphi_1 \sin \varphi_1 \right) z_1^2 + \left( \chi_{33} \cos^2 \varphi_2 + \chi_{44} \sin^2 \varphi_2 + 2\chi_{34} \cos \varphi_2 \sin \varphi_2 \right) z_2^2 - 2 \cos \varphi_1 \times \{ \chi_{13} \cos \varphi_2 + \chi_{14} \sin \varphi_2 \} + \sin \varphi_1 \{ \chi_{23} \cos \varphi_2 + \chi_{24} \sin \varphi_2 \} \right) \right] z_1 z_2]. \quad (4.17)$$

Then, the joint PDF $p_{\psi_1 \psi_2}(\varphi_1, \varphi_2)$ of the random phases $\psi_1$ and $\psi_2$ can be obtained by solving the integrals over the joint PDF $p_{R_1, R_2 \psi_1 \psi_2}(R_1, R_2, \varphi_1, \varphi_2)$ as follows

$$p_{\psi_1 \psi_2}(\varphi_1, \varphi_2) = \int_0^\infty \int_0^\infty p_{R_1, R_2 \psi_1 \psi_2}(z_1, z_2, \varphi_1, \varphi_2) dz_1 dz_2. \quad (4.18)$$

The double integral involved in (4.18) can be solved by making use of the multiple integrals evaluation approach, which has previously been described in Section 3.2 of chapter 3. The application of this approach yields the following closed-form expression for the joint PDF $p_{\psi_1 \psi_2}(\varphi_1, \varphi_2)$ of the phases $\psi_1$ and $\psi_2$

$$p_{\psi_1 \psi_2}(\varphi_1, \varphi_2) = \frac{B_1}{4B_2^2 (E_1 F_1 - D_1^2)} \times \left( 1 + \frac{D_1}{(E_1 F_1 - D_1^2)^{1/2}} \times \left[ \frac{\pi}{2} + \tan^{-1} \left( \frac{D_1}{(E_1 F_1 - D_1^2)^{1/2}} \right) \right] \right) \quad (4.19)$$

where the quantities $D_1$, $E_1$, and $F_1$ are given by

$$D_1 = \cos \varphi_1 \{ \chi_{13} \cos \varphi_2 + \chi_{14} \sin \varphi_2 \} + \sin \varphi_1 \{ \chi_{23} \cos \varphi_2 + \chi_{24} \sin \varphi_2 \}$$
$$E_1 = \chi_{11} \cos^2 \varphi_1 + \chi_{22} \sin^2 \varphi_1 + 2\chi_{12} \cos \varphi_1 \sin \varphi_1$$
$$F_1 = \chi_{33} \cos^2 \varphi_2 + \chi_{44} \sin^2 \varphi_2 + 2\chi_{34} \cos \varphi_2 \sin \varphi_2. \quad (4.20)$$

It can be verified that, for $q = 1$ and $\Delta \phi = 0$, i.e., the case corresponding to the transmission of an unmodulated over the Rayleigh fading channels, by using the help of [Gradshteyn 1994,
4.2 Simulation and results verification

eq.1.624 (2)] and doing algebraic manipulations, (4.19) simplifies to

\[ p_{\psi_1 \psi_2}(\varphi_1, \varphi_2) = \frac{1 - \rho_{\mu_1+n}^2}{4\pi^2 (1 - (\rho_{\mu_1+n} \cos (\varphi_2 - \varphi_1))^2)} \left[ 1 + \frac{\rho_{\mu_1+n} \cos (\varphi_2 - \varphi_1)}{\sqrt{1 - (\rho_{\mu_1+n} \cos (\varphi_2 - \varphi_1))^2}} \right] \times \left( \frac{\pi}{2} + \sin^{-1} (\rho_{\mu_1+n} \cos (\varphi_2 - \varphi_1)) \right) \]  

(4.21)

where the quantity \( \rho_{\mu_1+n} \) represents the normalized ACF of the process \((\mu_1(t) + n_1(t))\) and is given by

\[ \rho_{\mu_1+n} = \left( \frac{\Gamma_{\mu_1\mu_1}(\tau) + \Gamma_{nn}(\tau)}{\sigma_{11}^2 + \sigma_n^2} \right). \]  

(4.22)

It should be noted that the simplified joint PDF is in agreement with the already known result [Bramley 1951, eq. (33)]. To the best of our knowledge, (4.19) is new and it constitutes the basis for the determination of the PDF \( p_{\Delta \Omega}(\varphi) \). In fact, using (4.19) and noting that \( \psi_2 - \psi_1 = \Delta \phi + \Delta \Omega \), allows us to obtain the desired PDF \( p_{\Delta \Omega}(\varphi) \) of the phase difference \( \Delta \Omega \) according to

\[ p_{\Delta \Omega}(\varphi) = \int_{-\pi}^{\pi} p_{\psi_1 \psi_2}(\varphi_1, \varphi_1 + \Delta \phi + \varphi) d\varphi_1. \]  

(4.23)

Unfortunately, the finite range integration involved in (4.23) is difficult to handle, and it can be evaluated only by using numerical techniques. For the special case given by \( q = 1 \), i.e., the Rayleigh channel case, the underlying integral is easily solved and it is found that (4.23) is in agreement with the result [Adachi 1992, eq.(4)]. Furthermore, for \( q = 1 \) and \( \Delta \phi = 0 \), (4.23) simplifies to

\[ p_{\Delta \Omega}(\varphi) = \frac{1 - \rho_{\mu_1+n}^2}{2\pi (1 - (\rho_{\mu_1+n} \cos (\varphi))^2)} \left[ 1 + \frac{\rho_{\mu_1+n} \cos (\varphi)}{\sqrt{1 - (\rho_{\mu_1+n} \cos (\varphi))^2}} \right] \left( \frac{\pi}{2} + \sin^{-1} (\rho_{\mu_1+n} \cos (\varphi)) \right) \]  

(4.24)

which corresponds perfectly to [Bramley 1951, eq. (34)].

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In this section, we check the validity of the derived PDF \( p_{\Delta \Omega}(\varphi) \) based on the computer simulation results obtained for the case of a M2M single Hoyt fading channel. For this channel type, as it has previously been reviewed in chapter 2, the ACF \( \Gamma_{\mu_1\mu_1}(\tau) \) of the process \( \mu_1(t) \)
4.2. Simulation and results verification

\[
\cos \left[ 2\pi \left( f_{T,1}^i + f_{R,1}^i \right) t + \theta_{i,1} \right] \quad c_{i,1}
\]

\[
\cos \left[ 2\pi \left( f_{T,2}^i + f_{R,2}^i \right) t + \theta_{i,2} \right] \quad c_{i,2}
\]

\[
\cos \left[ 2\pi \left( f_{T,N_i}^i + f_{R,N_i}^i \right) t + \theta_{i,N_i} \right] \quad c_{i,N_i}
\]

\[
\tilde{\mu}_{1i}(t)
\]

Figure 4.3: Deterministic simulation model for the process \( \mu_{1i}(t) \).

\((i = 1, 2)\) is given by [Akki 1986]

\[
\Gamma_{\mu_{1i},\mu_{1i}}(\tau) = \sigma_{1i}^2 J_0(2\pi f_{T,\text{max}} \tau) J_0(2\pi f_{R,\text{max}} \tau).
\]  

\[(4.25)\]

The simulation method used to generate the M2M single Hoyt fading channel, i.e., the single Hoyt fading with double-Doppler PSD, is based on the concept of Rice’s sum-of-sinusoids [Rice 1944, Rice 1945]. According to that principle, the process \( \mu_{1i}(t) \) is approximated by a finite sum of weighted and properly designed sinusoids. The resulting process represents the so-called deterministic process \( \tilde{\mu}_{1i}(t) (i = 1, 2) \) given by [Hajri 2005]

\[
\tilde{\mu}_{1i}(t) = \sum_{n=1}^{N_i} c_{i,n} \cos \left[ 2\pi \left( f_{T,n}^i + f_{R,n}^i \right) t + \theta_{i,n} \right]
\]

\[(4.26)\]

where \( N_i \) denotes the number of sinusoids used for the generation of \( \tilde{\mu}_{1i}(t) \) \((i = 1, 2)\), \( c_{i,n} \) are the gains, and \( f_{T,n}^i \) and \( f_{R,n}^i \) are the discrete Doppler frequencies caused by the motion of the transmitter and receiver, respectively. Also, in (4.26), \( \theta_{i,n} \) stands for the phases which are uniformly distributed in the interval \([0, 2\pi]\). In Figure 4.3, we show the general structure of the deterministic simulation process \( \tilde{\mu}_{1i}(t) \) in its continuous-time representation. For the computation of the gains and discrete Doppler frequencies of the underlying deterministic model,
we apply the so-called mean square error method (MSEM) [Hajri 2005, Pätzold 2002]. Following this deterministic method, the quantities \(c_{1,n}, c_{2,n}, f_{T,n}^i,\) and \(f_{R,n}^i\) are given by [Hajri 2005]

\[
c_{1,n} = 2\sigma_1 \left\{ \frac{1}{\tau_{max_1}} \int_0^{\tau_{max_1}} J_0 \left( 2\pi f_{T,max} \tau \right) J_0 \left( 2\pi f_{R,max} \tau \right) \cos \left[ 2\pi \left( f_{T,n} + f_{R,n} \right) \tau \right] d\tau \right\}^{1/2}
\]

\[
c_{2,n} = 2q \cdot \sigma_1 \left\{ \frac{1}{\tau_{max_2}} \int_0^{\tau_{max_2}} J_0 \left( 2\pi f_{T,max} \tau \right) J_0 \left( 2\pi f_{R,max} \tau \right) \cos \left[ 2\pi \left( f_{T,n}^2 + f_{R,n}^2 \right) \tau \right] d\tau \right\}^{1/2}
\]

\[
f_{T,n}^i = \frac{f_{T,max}}{2N_i} (2n - 1)
\]

\[
f_{R,n}^i = \frac{f_{R,max}}{2N_i} (2n - 1),
\]

respectively, where \(\tau_{max_i} = \frac{N_i}{2 [f_{T,max} + f_{R,max}]}\) \((i = 1, 2)\). Concerning the correlation of the Gaussian noise components \(n_1(t)\) and \(n_2(t)\), we consider an ACF \(\Gamma_{nn}(\tau)\) of a Gaussian filter with a bandwidth \(B\). In this case, \(\Gamma_{nn}(\tau)\) can be expressed as

\[
\Gamma_{nn}(\tau) = \sigma_n^2 \exp \left[ - (\pi B \tau)^2 \right].
\]

In Figure 4.4, we show the theoretical PDF \(p_{\Delta \Omega}(\varphi)\) of the phase difference \(\Delta \Omega\), along with the corresponding simulation data, for \(q = 0.5, \Lambda = 10\) dB, \(f_{T,\text{max}} = 20\) Hz, \(f_{R,\text{max}} = 30\) Hz, \(N_1 = 10, N_2 = 11,\) and various values of the ACF \(\Gamma_{nn}(\tau)\). First, it can be observed from this figure that there exists a reasonable mutual agreement between theory and simulation for all cases. Second, the largest spreading and the smallest maxima of the PDF \(p_{\Delta \Omega}(\varphi)\) is obtained for \(BT \geq 1(\Gamma_{nn}(T) \simeq 0)\), i.e., the case of uncorrelated Gaussian noise. A comparison between the theoretical PDF \(p_{\Delta \Omega}(\varphi)\) and the corresponding simulation one is shown in Figure 4.5, for \(\Lambda = 10\) dB, \(f_{T,\text{max}} = 20\) Hz, \(f_{R,\text{max}} = 30\) Hz, \(\Gamma_{nn}(T) = \exp[-\pi^2], N_1 = 10, N_2 = 11,\) and different values of the Hoyt fading parameter \(q\). A good agreement can be noted between theoretical and simulation results. In Figure 4.6, we present a comparison between the analytical and simulation PDF \(p_{\Delta \Omega}(\varphi)\) for \(q = 0.5, f_{T,\text{max}} = 20\) Hz, \(f_{R,\text{max}} = 30\) Hz, \(\Gamma_{nn}(T) = \exp[-\pi^2], N_1 = 10, N_2 = 11,\) and three values of the parameter \(\Lambda\). As can be seen, the largest spreading of \(p_{\Delta \Omega}(\varphi)\) is obtained for the minimum value of \(\Lambda, i.e., \Lambda = 10\) dB. However, the highest maxima of the PDF \(p_{\Delta \Omega}(\varphi)\) corresponds to the maximum value of \(\Lambda, i.e., \Lambda = 30\) dB. The effect of the maximum Doppler frequencies \(f_{T,\text{max}}\) and \(f_{R,\text{max}}\) on the PDF \(p_{\Delta \Omega}(\varphi)\), for \(q = 0.5, \Lambda = 10\) dB, \(\Gamma_{nn}(T) = \exp[-\pi^2], N_1 = 10, N_2 = 11,\) can be studied from Figure 4.7. Again, from this figure, a good fit between the theoretical and simulation results can be observed.
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Figure 4.4: The PDF $p_{\Delta \Omega}(\varphi)$ for various values of the ACF $\Gamma_{nn}(T)$.

Figure 4.5: The PDF $p_{\Delta \Omega}(\varphi)$ for various values of the Hoyt fading parameter $q$. 

$\omega$ = 0.5
$\Lambda = 10$ dB
$f_{T,\text{max}} = 20$ Hz
$f_{R,\text{max}} = 30$ Hz
$N_1 = 10$
$N_2 = 11$
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Figure 4.6: The PDF $p_{\Delta \Omega}(\varphi)$ for various values of the parameter $\Lambda$.

Figure 4.7: The PDF $p_{\Delta \Omega}(\varphi)$ for different combinations of the maximum Doppler frequencies $f_{T,\text{max}}$ and $f_{R,\text{max}}$. 

$q = 0.5$

$\Lambda = 10$ dB

$N_1 = 10$

$N_2 = 11$
4.3 Conclusion

In this chapter, a closed-form expression for the PDF of the phase difference between two Hoyt faded signals perturbed by the correlated Gaussian noise has been derived. The obtained PDF expression is verified to reduce to known results corresponding to the Rayleigh fading as a special case of the Hoyt fading model. Furthermore, the validity of the presented results has been checked by computer simulations, obtained for a M2M single Hoyt fading channel. The newly derived PDF formula can then be applied in the evaluation of the error rate performance of wireless M2M communications over the single Hoyt multipath fading channels with a double-Doppler PSD.

Specifically, based on this PDF expression and drawing upon the theory on the BEP, the error rate performance of the DPSK modulation schemes and FSK with the LDI and differential detections are investigated. Much more details on the derivation of analytical expressions for the BEP performance of the DPSK and FSK modulation schemes over the single Hoyt fading channels with a double-Doppler PSD, i.e., the so-called M2M single Hoyt fading channels, will be presented in the next chapter.
Chapter 5

**BEP Performance of DPSK and FSK modulation schemes over M2M single Hoyt fading channels**

Based upon the newly derived PDF formula for the phase difference between two Hoyt vectors contaminated by AWGN, which has been presented in the previous chapter, and the theory of digital modulations together with that of the error rate performance, the BEP performance of the DPSK modulation schemes and FSK with the LDI and differential detections are investigated and analyzed. Namely, analytical expressions for the BEP performance of all these modulation schemes over the Hoyt multipath fading channels, taking into account the Doppler spread effects caused by the motion of the mobile transmitter and receiver, are derived.

The objective of this chapter is to present all details on the derivation of these analytical BEP expressions, considering the case of M2M single Hoyt fading channels, i.e., the case of Hoyt multipath fading channels with a double-Doppler PSD.

The remainder of the chapter is organized as follows. In Section 5.1, a study on the BEP performance of the DPSK modulation schemes over frequency-flat M2M single Hoyt fading channels is presented. The performance analysis of the FSK modulation with an LDI detection over the same M2M channels is studied, in Section 5.2. The BEP of the one-bit delay differential detection of narrowband FSK signals transmitted over M2M single Hoyt fading channels is provided, in Section 5.3. Finally, Section 5.4 concludes the chapter.

5.1 **Performance analysis of DPSK modulation schemes**

The objective of this section is to provide an analysis on the BEP performance of the DPSK modulation over the so-called M2M single Hoyt fading channels. The derivation of an analytical expression for the corresponding BEP is carried out by applying the theory of the DPSK modulation [Voelcker 1960, Miyagaki 1979], and using the recently derived PDF formula in chapter
4. The obtained results are then validated by the means of computer simulations.

5.1.1 DPSK receiver

The binary DPSK receiver under study is shown, as a block diagram, in Figure 5.1. The phase modulated signal is assumed to be transmitted over a frequency-nonselective Hoyt fading channel. After the propagation through the Hoyt fading channel, the received signal is perturbed by an AWGN and bandlimited by a rectangular bandpass filter with an equivalent bandwidth \( B \).

This pre-detection filter is assumed to be an ideal bandpass filter which satisfies the "Nyquist filter" criterion. In this case, the bandwidth-time product \( BT \) coefficient, with \( T \) denoting the one bit duration, is given by \( BT \geq 1 \) [Proakis 2001]. Based on this assumption, no ISI are introduced at the output of the ideal rectangular bandpass filter. Therefore, the resultant noise-corrupted fading signal, at the output of the above filter, can be described mathematically by

\[
s_0(t) = A(t) \exp[j\psi(t)]
\] (5.1)

where \( A(t) \) denotes the envelope signal and \( \psi(t) = \phi(t) + \Omega(t) \) stands for the overall phase of the received signal as it has been mentioned in chapter 4. During any bit interval \( T \), the data phase \( \phi(t) \) is either 0 or \( \pi \), depending upon whether a space ("0") or a mark ("1") is being transmitted, respectively. Now, the DPSK receiver delays the pre-detection filter output signal \( s_0(t) \) by the one bit duration \( T \) to yield \( s_0(t - T) \). Then, it multiplies the real part of \( s_0(t) \), i.e., \( \text{Re}\{s_0(t)\} \), with its real delayed version \( \text{Re}\{s_0(t - T)\} \) resulting, therefore, in the following quantity

\[
s_1(t) = \text{Re}\{s_0(t)\} \times \text{Re}\{s_0(t - T)\}
\]

\[
= \frac{A(t)A(t - T)}{2} \left\{ \cos(\psi(t) - \psi(t - T)) + \cos(\psi(t) + \psi(t - T)) \right\}.
\] (5.2)
Thereafter, the obtained signal in (5.2) is fed into a lowpass filter. After filtering the high frequency terms, the post-detection filter outputs the signal given by

\[ s_d(t) = \frac{A(t)A(t-T)}{2} \cos(\Delta \psi) \]  \hspace{1cm} (5.3)

where \( \Delta \psi = \psi(t) - \psi(t-T) \) stands for the overall phase difference and can be expressed as

\[ \Delta \psi = \phi(t) - \phi(t-T) + \Omega(t) - \Omega(t-T) = \Delta \phi + \Delta \Omega. \] \hspace{1cm} (5.4)

Finally, the filtered output signal \( s_d(t) \) is sampled and fed into a decision device. Since the envelope \( A(t) \) is always positive, the decision on whether a mark or a space is sent is essentially based on the polarity of the quantity \( \cos(\Delta \psi) \). Therefore, in order to analyze the error rate performance of the binary DPSK modulation scheme, it is necessary to determine the PDF \( p_{\Delta \psi}(\varphi) \) of the phase difference \( \Delta \psi \). An expression for this PDF is given by

\[ p_{\Delta \psi}(\varphi) = \int_{-\pi}^{\pi} p_{\psi_1,\psi_2}(\varphi_1, \varphi_1 + \varphi) d\varphi_1 \] \hspace{1cm} (5.5)

where \( p_{\psi_1,\psi_2}(\cdot, \cdot) \) is the joint PDF of the random phases \( \psi_1 = \phi(t-T) + \Omega(t-T) \) and \( \psi_2 = \phi(t) + \Omega(t) \). An expression for this joint PDF has been derived in chapter 4, and is given by (4.19). The PDF \( p_{\Delta \psi}(\varphi) \) of the phase difference \( \Delta \psi \), is valid for any Doppler PSD, especially for the Doppler PSD of the M2M Hoyt fading channels. In Figure 5.2, we show the PDF \( p_{\Delta \psi}(\varphi) \) of the phase difference \( \Delta \psi \), for \( BT = 1.0, \Delta \phi = \pi/3, \Lambda = 10 \text{ dB}, f_{T,\text{max}} \cdot T = f_{R,\text{max}} \cdot T = 0.004 \), and various values of the Hoyt fading parameter \( q \). As it can be seen, the PDF is symmetrical about \( \Delta \phi = \pi/3 \). In addition, the largest spreading of the PDF \( p_{\Delta \psi}(\varphi) \) is obtained for \( q = 0 \), i.e., the case of the one-sided Gaussian channel. However, the smallest one corresponds to the Rayleigh fading channel, i.e., the case when \( q = 1 \).

### 5.1.2 Bit error probability

The average BEP performance of the DPSK modulation with a noncoherent detection can be obtained according to [Lee 1975, Pawula 1984]

\[ P_E = p_m P_E (M) + (1 - p_m) P_E (S) \] \hspace{1cm} (5.6)
Figure 5.2: The PDF $p_{\Delta \psi}(\phi)$ of the overall phase difference $\Delta \psi$ for various values of the Hoyt fading parameter $q$.

where $p_m$ is the probability of mark in the information signal, while $P_E(M)$ and $P_E(S)$ are the conditional probabilities of error, given the transmission of mark and space, respectively. Following [Lee 1975, Pawula 1984], these conditional probabilities can be calculated as

$$P_E(M) = \text{Prob}(\cos(\Delta \psi) > 0|\Delta \phi = \pi)$$  \hspace{1cm} (5.7)

and

$$P_E(S) = \text{Prob}(\cos(\Delta \psi) < 0|\Delta \phi = 0)$$  \hspace{1cm} (5.8)

where $\text{Prob}(\cos(\Delta \psi) > 0|\Delta \phi = \pi)$ and $\text{Prob}(\cos(\Delta \psi) < 0|\Delta \phi = 0)$ are the conditional error probabilities given that $\Delta \phi = \pi$ and $\Delta \phi = 0$ when $\cos(\Delta \psi) > 0$ and $\cos(\Delta \psi) < 0$, respectively. Now, it is obvious that the determination of $P_E(M)$ and $P_E(S)$ can be carried out by applying the PDF $p_{\Delta \psi}(\phi)$ of the phase difference $\Delta \psi$. Specifically, using [Pawula 1984] and (5.5), these quantities can be obtained as

$$P_E(M) = \int_{-\pi/2}^{\pi/2} p_{\Delta \psi}(\phi|\Delta \phi = \pi)d\phi$$  \hspace{1cm} (5.9)
5.1. Performance analysis of DPSK modulation schemes

\[ P_E (S) = \int_{\pi/2}^{3\pi/2} p_{\Delta \psi}(\varphi | \Delta \phi = 0) d\varphi. \]  \hspace{1cm} (5.10)

Unfortunately, the finite range integrations in (5.9) and (5.10) can be evaluated only by numerical techniques. Finally, the desired BEP performance of the DPSK detection over the Hoyt fading channels can be obtained by substituting (5.9) and (5.10) in (5.6). It should be noted that for the special case when \( q = 1 \), i.e., the case of Rayleigh fading channels, the evaluation of (5.6) using (5.9) and (5.10) leads to

\[ P_E = \frac{1}{2} \left( 1 - \frac{\Gamma_{\mu_1 \mu_1}(T) + \Gamma_{mn}(T)}{\sigma_{11}^2 + \sigma_n^2} + 2p_m \frac{\Gamma_{nn}(T)}{\sigma_{11}^2 + \sigma_n^2} \right) \]  \hspace{1cm} (5.11)

which corresponds to the result reported in [Jakes 1993, Voelcker 1960].

5.1.3 Results verification

5.1.3.1 Simulation model

According to (2.1), the Hoyt channel gain can be simulated by the generation of two Gaussian random processes. The simulation method used here to generate the process (2.1) is based on the concept of Rice’s sum-of-sinusoids [Rice 1944, Rice 1945]. Following this concept, the Gaussian process \( \mu_1(t) \) \((i = 1, 2)\) describing the Hoyt channel gain process \( \mu_1(t) \) can be basically approximated using the deterministic process \( \tilde{\mu}_1(t) \) as expressed in (4.26). Again, for the computation of the parameters of the simulation model (4.26), i.e., the gains \( c_{i,n} \), the discrete frequencies \( f_{T,n}^{i} \) and \( f_{R,n}^{i} \), as well as the discrete phases \( \theta_{i,n} \), we use the MSEM method [Hajri 2005, Pätzold 2002]. These parameters have to be determined such that the ACF \( \Gamma_{\mu_1 \mu_1}(\tau) \) of the reference M2M single Hoyt fading channel, which is described by (4.25), is well approximated by that of the deterministic simulation model \( \tilde{\Gamma}_{\mu_1 \mu_1}(\tau) \) given by [Hajri 2005]

\[ \tilde{\Gamma}_{\mu_1 \mu_1}(\tau) = \sum_{n=1}^{N_i} \frac{2}{c_{i,n}^2} \cos \left[ 2\pi \left( f_{T,n}^{i} + f_{R,n}^{i} \right) \tau \right]. \]  \hspace{1cm} (5.12)

According to this deterministic method, the gains \( c_{i,n} \) and the discrete frequencies \( f_{T,n}^{i} \) and \( f_{R,n}^{i} \) \((i = 1, 2)\) are given by (4.27). In Figure 5.3, we show a comparison between the ACF \( \Gamma_{\mu_1 \mu_1}(\tau) \) of the reference model \( \mu_1(t) \) and that of the corresponding simulation model \( \tilde{\Gamma}_{\mu_1 \mu_1}(\tau) \), for \( q = 0.5, N_2 = 7, f_{T,max} = 80 \text{ Hz}, \) and \( f_{R,max} = 20 \text{ Hz}. \) As it can be seen, the theoretical results are well supported by the corresponding simulation ones over the range \([0, \tau_{max} = 0.035]\). Both
5.1. Performance analysis of DPSK modulation schemes

Figure 5.3: A comparison between the ACF $\Gamma_{\mu_{12}\mu_{12}}(\tau)$ of the M2M single Hoyt reference model and that of deterministic model $\Gamma_{\tilde{\mu}_{12}\tilde{\mu}_{12}}(\tau)$.

processes $\mu_{11}(t)$ and $\mu_{12}(t)$ are simulated according to the method that is described above except that $N_1$ should be taken differently from $N_2$, i.e., $N_1 \neq N_2$, in order to ensure a value zero for the cross-correlation between these processes.

5.1.3.2 Numerical and simulation examples

In this section, we present numerical results along with the corresponding simulation data for a comparison purpose. The bit-energy-to-noise ratio $E_b/N_0$ against which the BEP performance is plotted is related to the parameters $q$ and $\Lambda$ according to

$$E_b/N_0 = \frac{(1 + q^2)BT}{2} \Lambda$$

(5.13)

where $E_b$ stands for the average received signal energy per bit. The ACF of the baseband correlated Gaussian noise, present at the output of the ideal pre-detection filter, is given by

$$\Gamma_{nn}(\tau) = \sigma_n^2 \sin(\pi B\tau)/\pi B\tau.$$  

(5.14)

To describe the varying rate of the channel, we introduce the parameter $\alpha$ defined by

$$\alpha = f_{R,\max}/f_{T,\max}$$

(5.15)
with $0 \leq \alpha \leq 1$, assuming $f_{R,\text{max}} \leq f_{T,\text{max}}$. First, we consider the effect of the Hoyt fading severity parameter $q$ on the BEP performance. The relevant performance is shown in Figure 5.4, where the BEP is depicted as a function of $E_b/N_0$ for $BT = 1$, $f_{T,\text{max}} = 20$ Hz, $\alpha = 1$, and various values of $q$. The theoretical results are in a good agreement with the simulated ones, demonstrating, thus, the validity of the theoretical analysis. As expected, the lowest BEP is obtained for $q = 1$, i.e., the case of Rayleigh fading channels, while the highest one corresponds to the one-sided Gaussian channel, i.e., $q = 0$. It is shown that the BEP performance degrades with the decreasing values of $q$. For large values of $E_b/N_0$, an irreducible error floor, caused by the Doppler spread, appears. Next, the effect of the maximum Doppler frequencies on the BEP performance is examined. The illustrating curves are plotted in Figure 5.5, for $q = 0.6$, $BT = 1$, $f_{T,\text{max}} = 80$ Hz, and different values of $\alpha$. In this case, again, the simulated curves closely match the theoretical curves. The occurrence of the error floor at high values of $E_b/N_0$ can also be noticed from this figure. This error floor increases with the increase of the value of $\alpha$. Finally, for the sake of comparison, Figure 5.6 shows the BEP for the M2M Doppler PSD together with that obtained for the Jake’s Doppler PSD [Jakes 1993], i.e., the Doppler PSD corresponding to the conventional mobile cellular links. In the illustration, the maximum Doppler frequency encountered in the cellular radio links ($\alpha = 0$) is chosen to be equal to the sum of the maximum Doppler frequencies attributed to the motion of the transmitter and the
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Figure 5.5: Theoretical and simulated BEP performance of binary noncoherent DPSK for different values of the ratio of the maximum Doppler frequencies $\alpha$.

Figure 5.6: Theoretical and simulated BEP performance of binary noncoherent DPSK over conventional ($\alpha = 0$) and M2M ($\alpha = 1$) channels for different values of the Doppler frequency $f_{T,\text{max}}$. 
receiver in the M2M communication scenarios ($\alpha = 1$). As it can be noticed for this specific example, the results reveal that the BEP performance of the M2M radio links is better than that corresponding to the mobile cellular links.

5.2 Performance analysis of FSK with LDI detection

5.2.1 LDI receiver

The LDI based digital FM receiver is depicted, as a block diagram, in Figure 5.7. Again, the link between the transmitter and receiver is modeled by a narrowband Hoyt multipath fading channel. The complex lowpass equivalent Hoyt faded FSK signal, present at the input of the Gaussian IF pre-detection filter, can be expressed as

$$e(t) = \mu_1(t) \exp[j\theta(t)] + w(t)$$

where $w(t)$ denotes a zero-mean complex AWGN and $\theta(t)$ stands for the data phase after an FM modulation, which is given by (3.2). For the determination of the signal resulting at the output of the IF filter, i.e., $e_0(t)$, we follow Tjhung et al. [Tjhung 1990, Ng 1994] by assuming that the Hoyt fading process changes at a rate that is much slower than the data rate $1/T$. This so-called “quasi-static” analysis implies that the complex channel gain process $\mu_1(t)$, i.e., $\mu_1(t) + j\mu_2(t)$, is not affected by its passage over the IF filter. In this case, the output of the pre-detection filter can be written as

$$e_0(t) = \mu_1(t)a(t) \exp[j\phi(t)] + n_1(t) + jn_2(t)$$

where, as it has previously been mentioned, $a(t)$ and $\phi(t)$ are the IF filtered carrier amplitude and information phase, respectively, while $n_1(t)$ and $n_2(t)$ stand for the quadrature components of the IF filtered complex AWGN $w(t)$. The output signal of the LDI detector is the phase difference $\Delta\Psi$, over the bit time interval $[t - T, t]$, of the IF filtered FSK signal. As it is
discussed in chapter 3, this phase difference is given by [Pawula 1981]

\[ \Delta \Psi = \Delta \phi + \Delta \Omega + 2\pi N(t - T, t) \]  \hspace{1cm} (5.18)

where \( N(t - T, t) \) stands for the number of FM clicks occurring in the time interval \([t - T, t]\).

From this and based on [Pawula 1981], the probability of making an error, when a “+1” symbol
is sent, is obtained by computing the quantity \( \text{Prob}(\Delta \Psi \leq 0) \). Under the assumption of a
statistical independence between \( \Delta \Omega \) and \( N(t - T, t) \), this probability can be approximated by
(3.10), as

\[ \text{Prob}(\Delta \Psi \leq 0) = \text{Prob}(\Delta \Omega > \Delta \phi) + \overline{N} \]  \hspace{1cm} (5.19)

An expression of the average number of the positive FM clicks, \( \overline{N} \), occurring in the time interval
\([t - T, t]\) and taking into account the Hoyt fading effects has been derived in chapter 3 and can
be written as

\[ \overline{N} = \frac{1}{2\pi qa} \int_{t-T}^{t} \frac{\dot{\phi}(\tau)}{\sqrt{(a^2(\tau) + \frac{1}{\sigma^2}) (a^2(\tau) + \frac{1}{\Lambda})}} d\tau. \]  \hspace{1cm} (5.20)

The probability \( \text{Prob}(\Delta \Omega > \Delta \phi) \), which is required for the evaluation of the BEP according to
(5.19), can be computed from the PDF \( p_{\Delta \Omega}(\varphi) \) of the phase difference \( \Delta \Omega \) as follows

\[ \text{Prob}(\Delta \Omega > \Delta \phi) = \int_{\Delta \phi}^{\pi} p_{\Delta \Omega}(\varphi) d\varphi. \]  \hspace{1cm} (5.21)

In the following, we present the closed-form formula for the BEP performance of FSK with LDI
detection over the Hoyt mobile fading channels.

5.2.2 Bit error probability

In this section, we apply the newly derived PDF \( p_{\Delta \Omega}(\varphi) \) of the phase difference \( \Delta \Omega \), in the eval-
uation of the desired BEP. According to [Pawula 1981, Tjhung 1990, Ng 1994], the corresponding
BEP performance is obtained as

\[ P_E = \text{Prob}(\Delta \Psi \leq 0) = P_{e,1} + P_{e,2} \]  \hspace{1cm} (5.22)

where, for the case of absence of ISI, \( P_{e,1} = \text{Prob}(\Delta \Omega > \Delta \phi) \), which can be computed using
(5.21), whereas \( P_{e,2} = \overline{N} \), which is given directly by (5.20). Now, to take into consideration the
effect of the ISI caused by the bandwidth limitation of the IF filter, we follow [Pawula 1981] by
5.2. Performance analysis of FSK with LDI detection

Figure 5.8: BEP performance of the FSK modulation scheme with an LDI detection for various values of the Hoyt fading parameter $q$.

assuming a time-bandwidth product $BT \geq 1$. In this case, as it has been discussed in chapter 3, only the bits adjacent to that being detected are taken into account. Namely, when a “+1” symbol is sent, only the three bit patterns given by “111”, “010” and “011” are considered in the ISI evaluation. Then, by considering the ISI effects, the quantities $P_{e,1}$ and $P_{e,2}$ needed for the calculation of the BEP according to (5.22), are given by (3.40) and (3.41) respectively. For all the three bit patterns considered in the analysis, expressions for the quantities $\Delta \phi$, $\dot{\phi}(t)$, and $a^2(t)$, which are needed in the evaluation of (5.22), can be found in the Appendix A.

5.2.3 Numerical examples

In this section, computed numerical results for the closed-form BEP formula given by (5.22) are presented for the Doppler PSD of a M2M single Hoyt fading channel. Concerning the additive Gaussian noise, we assume uncorrelated noise samples, i.e., $\Gamma_{nn}(T) = 0$. The bit-energy-to-noise ratio $E_b/N_0$ against which the BEP is plotted is related to the parameters $q$ and $\Lambda$ according to (5.13). Figure 5.8 shows the effect of the Hoyt fading parameter $q$ on the BEP, when $BT = 1.0$, $f_{T,\text{max}} = f_{R,\text{max}} = 40$ Hz, $T = 10^{-4}$ s, and the modulation index $h = 0.7$. As expected, these results indicate that the BEP $P_E$ improves as $q$ increases. The best performance is obtained for $q = 1$, i.e., the case of the Rayleigh fading channel. The average BEP versus $E_b/N_0$ is depicted, in Figure 5.9, for $q = 0.4$, $h = 0.7$, $f_{T,\text{max}} = f_{R,\text{max}} = 40$ Hz, $T = 10^{-4}$ s, and three
5.2. Performance analysis of FSK with LDI detection

Figure 5.9: BEP performance of the FSK modulation scheme with an LDI detection for various values of $BT$.

Figure 5.10: BEP performance of the FSK modulation scheme with an LDI detection for various combinations of the maximum Doppler frequencies $f_{T,max}$ and $f_{R,max}$. 

$q = 0.4$
$h = 0.7$
$T = 10^{-4}$ s
$f_{T,max} = f_{r,max} = 40$ Hz

$q = 0.4$
$h = 0.7$
$BT = 1.0$
$T = 10^{-4}$ s
values of $BT$. It should be noted from this figure that before the appearance of the error floor, the BEP gets larger with increasing values of $BT$. However, as $E_b/N_0$ becomes very large, the BEP improves with increasing $BT$. This reversal effects of $BT$ on the BEP is due to the fact that as $BT$ gets small, $\Delta \phi$ decreases and, then, the irreducible BEP increases. The behavior of the average BEP $P_E$, as a function of $E_b/N_0$ for $q = 0.4$, $BT = 1.0$, $h = 0.7$, $T = 10^{-4}$ s, and various combinations of the maximum Doppler frequencies $f_{T,\text{max}}$ and $f_{R,\text{max}}$, can be studied from Figure 5.10. As can be seen, the effect of the Doppler frequencies appears only at high values of $E_b/N_0$. Furthermore, it is worth noting that the values of the error floor for a conventional communication, where only the transmitter or the receiver is in motion (i.e., the example of $f_{T,\text{max}} = 40$ Hz and $f_{R,\text{max}} = 0$ Hz in Figure 5.10), are a little higher than those corresponding to the case of a M2M communication where we have the same combined value of the maximum Doppler frequencies, i.e., $f_{T,\text{max}} = f_{R,\text{max}} = 20$ Hz.

5.3 Performance analysis of FSK with differential detection

The aim of this part is to contribute to the topic of the performance analysis of the differential detection of FSK modulation, over Hoyt fading channels, by studying the corresponding BEP. Again, we rely, in the study, on the classical theory of the differential detection [Simon 1983] and the determined expression, in chapter 4, for the PDF of the phase difference of Hoyt faded signals corrupted by the additive Gaussian noise.

5.3.1 Differential receiver

In Figure 5.11, we show, as a block diagram, the FSK one-bit delay differential receiver. Following the so-called “quasi-static” analysis mentioned above, the filtered baseband Hoyt faded FSK signal, resulting at the output of the Gaussian IF Filter, can be written as

$$e_0(t) = e_{01}(t) + je_{02}(t)$$

$$= A(t) \cos(\phi(t) + \Omega(t)) + jA(t) \sin(\phi(t) + \Omega(t))$$ (5.23)
where \( e_{01}(t) = A(t) \cos(\phi(t) + \Omega(t)) \) denotes the real component of \( e_0(t) \) and \( e_{02}(t) = A(t) \sin(\phi(t) + \Omega(t)) \) represents the imaginary component. By considering the real part of the signal \( e_0(t) \), i.e., \( e_{01}(t) \), the differential detector first multiplies \( e_{01}(t) \) by a version of it that is delayed by the one-bit symbol time \( T \) and phase-shifted by \( \pi/2 \). Then, the high frequency terms are filtered out, and we get at the output of the differential detector the signal \( y(t) \) as

\[
y(t) = \frac{A(t)A(t-T)}{2} \sin(\Delta\psi)
\]

where the overall phase difference \( \Delta\psi \) is given by (5.4). Since the envelope \( A(t) \) is always positive, the receiver decides that a “+1” symbol is sent if \( \sin(\Delta\psi) > 0 \) and a “0” if \( \sin(\Delta\psi) \leq 0 \).

Therefore, to investigate the average BEP performance of the one-bit delay FSK differential detection in Hoyt fading channels we, again, need the PDF of the overall phase difference \( \Delta\psi \).

In the following, we present the BEP performance of the FSK modulation with differential detection in Hoyt fading channels.

5.3.2 Bit error probability

In this section, we will employ (5.5) to evaluate the average BEP for the one-bit delayed differential detection of the digital FSK signals transmitted over a Hoyt fading channel and corrupted by AWGN. Due to the bandwidth limitation of the pre-detection IF filter, ISI are introduced on the IF filtered FSK signals. To take into account these ISI in the evaluation of the BEP, we follow [Pawula 1981, Ng 1994] and suppose that only the bits that are adjacent to the bit being detected are of a significant degradation effect. Then, since the phase difference \( \Delta\psi \) is restricted to the modulo \( 2\pi \) and the choice of the \( 2\pi \) domain is arbitrary, it is convenient to choose this interval \([\Delta\phi - \pi, \Delta\phi + \pi]\) [Simon 1983]. Based on all these assumptions, the desired average BEP \( P_E \) of the differential detection scheme can be computed according to [Simon 1983]

\[
P_E = \frac{1}{4} \left[ \text{Prob}(\Delta\phi - \pi \leq \Delta\psi \leq 0|_{111}) + \text{Prob}(\pi \leq \Delta\psi \leq \pi + \Delta\phi|_{111}) \right. \\
+ \text{Prob}(\Delta\phi - \pi \leq \Delta\psi \leq 0|_{010}) + \text{Prob}(\pi \leq \Delta\psi \leq \pi + \Delta\phi|_{010}) \\
+ 2\text{Prob}(\Delta\phi - \pi \leq \Delta\psi \leq 0|_{011}) + 2\text{Prob}(\pi \leq \Delta\psi \leq \pi + \Delta\phi|_{011}) \right]
\] (5.25)

where \( \text{Prob}(\Delta\phi - \pi \leq \Delta\psi \leq 0|\ldots) \) and \( \text{Prob}(\pi \leq \Delta\psi \leq \pi + \Delta\phi|\ldots) \) are the conditional error probabilities given the bit pattern “...”, i.e., “111”, “010”, or “011”, when the phase difference \( \Delta\psi \) lies in the interval \([\Delta\phi - \pi, 0]\) and \([\pi, \Delta\phi + \pi]\), respectively. These probabilities are obtained by first inserting, in (5.25), the values of \( \Delta\phi \) corresponding to the occurrence of the bit patten
5.3. Performance analysis of FSK with differential detection

“...”, and then evaluating the integrals given by

\[
\text{Prob}(\Delta \phi - \pi \leq \Delta \psi \leq 0 | \ldots) = \int_{\Delta \phi - \pi}^{0} p_{\Delta \psi}(\varphi) d\varphi, \quad (5.26)
\]

and

\[
\text{Prob}(\pi \leq \Delta \psi \leq \pi + \Delta \phi | \ldots) = \int_{\pi}^{\Delta \phi + \pi} p_{\Delta \psi}(\varphi) d\varphi. \quad (5.27)
\]

The values of the quantity \(\Delta \phi\), for all the bit patterns “111”, “010”, and “011” considered in the analysis, can be found in the Appendix A.

5.3.3 Numerical examples

In this section, we present computed numerical results for the average BEP performance of the FSK differential detection scheme and compare it with that of the LDI detector for the case of the M2M single Hoyt fading channels. The numerical results for the differential detector receiver are computed from (5.25), while those corresponding to the LDI receiver are reproduced from (5.22). The average BEPs \(P_E\) of the differential and LDI detectors are depicted as a function of \(E_b/N_0\) ratio given by (5.13). Figure 5.12 shows the BEPs of the differential and LDI detection schemes versus \(E_b/N_0\) for \(h = 0.7, BT = 1.0, T = 10^{-4} \text{ s}, f_{T,\text{max}} = f_{R,\text{max}} = 40 \text{ Hz}\), and three values of \(q\). As expected, the best performance is obtained for \(q = 1.0\), i.e., the case of the Rayleigh fading channels. In addition, it should be noted from this figure that the LDI receiver gives a better performance than the differential detection receiver for all values of the Hoyt fading parameter \(q\). In Figures 5.13–5.15, we show the effects of the bandwidth-time product \(BT\) and the modulation index \(h\) on the BEP performance of the differential detection as compared with that of the LDI detection for \(q = 0.4, T = 10^{-4} \text{ s}, f_{T,\text{max}} = f_{R,\text{max}} = 40 \text{ Hz}\). From these figures, one can make several observations. First, the LDI receiver gives a superior performance than the differential receiver for all the considered values of \(BT\) and \(h\). Second, before the appearance of the error floor, for \(h = 0.5\) and all the values of \(BT\), the two receivers give almost the same BEP performance. As \(h\) increases the differences in performance become more and more significant, and for \(h = 1.0\) the differential detection yields very poorer performance with respect to that of the LDI detector. The behavior of the average BEP \(P_E\) of the differential detection receiver, compared with that of the LDI detector, for \(q = 0.4, BT = 1.0, h = 0.7, T = 10^{-4} \text{ s},\) and different values of \(f_{T,\text{max}}\) and \(f_{R,\text{max}}\), can be studied from Figure 5.16. It should be emphasized that all the above observations are in a perfect agreement
5.3. Performance analysis of FSK with differential detection

Figure 5.12: A comparison between the BEP of the FSK differential and LDI detectors for different values of the Hoyt fading parameter $q$.

Figure 5.13: A comparison between the BEP of the FSK differential and LDI detectors for $h = 0.5$ and different values of $BT$. 
5.3. Performance analysis of FSK with differential detection

Figure 5.14: A comparison between the BEP of the FSK differential and LDI detectors for $h = 0.7$ and different values of $BT$.

Figure 5.15: A comparison between the BEP of the FSK differential and LDI detectors for $h = 1.0$ and different values of $BT$. 
with the results that have been reported for the Rayleigh fading channels [Simon 1983].

5.4 Conclusion

In this chapter, an analytical expression for the BEP performance of noncoherent DPSK modulation schemes over the Hoyt fading channels, taking into account the Doppler spread effects caused by the motion of the mobile transmitter and receiver, has first been derived. The presented theoretical results have been verified to include the known result for the Rayleigh model as a special case of the Hoyt model. Furthermore, the validity of the theory has been confirmed by computer simulations performed for a M2M single Hoyt fading channel. Then, a performance analysis of a narrowband FSK modulation with LDI and differential detections has been presented, considering the M2M single Hoyt multipath fading channels. Numerical results for the corresponding BEPs performance have been presented for several values of the FM system parameters and the M2M Hoyt fading channel characteristics. From all the obtained BEP results, it has been demonstrated that the Hoyt fading channel leads to a poor performance when compared to the case of the Rayleigh fading conditions. In addition, the BEP performance degrades with decreasing values of the Hoyt fading severity parameter and, therefore, the worst performance corresponds to the one-sided Gaussian channel, i.e., $q = 0$. 

Figure 5.16: A comparison between the BEP of the FSK differential and LDI detectors for various combination of the maximum Doppler frequencies $f_{T,max}$ and $f_{R,max}$. 

\[ f_{T,max} = 40 \text{ Hz} \land f_{R,max} = 0 \text{ Hz}(\text{LDI}) \]
\[ f_{T,max} = 40 \text{ Hz} \land f_{R,max} = 0 \text{ Hz}(\text{Differential}) \]
\[ f_{T,max} = 20 \text{ Hz} \land f_{R,max} = 20 \text{ Hz}(\text{LDI}) \]
\[ f_{T,max} = 20 \text{ Hz} \land f_{R,max} = 20 \text{ Hz}(\text{Differential}) \]
\[ f_{T,max} = 40 \text{ Hz} \land f_{R,max} = 40 \text{ Hz}(\text{LDI}) \]
\[ f_{T,max} = 40 \text{ Hz} \land f_{R,max} = 40 \text{ Hz}(\text{Differential}) \]
As it has been mentioned in the introduction of this dissertation, in addition to the M2M single Hoyt fading channels, the double Hoyt fading model can also be used in the statistical description of more realistic M2M propagation environments. Besides, the importance of the double Hoyt fading, which can be used in the modelling of M2M fading channels where the fading conditions are worse than those described by the double Rayleigh fading case, there exists no investigation results on this cascaded channel model so far.

The objective of the next chapter is to contribute to the double Hoyt fading channels by studying and investigating their main statistical properties and their corresponding commonly used performance measures of wireless communication systems, e.g., frequency of outage and average outage duration.
Recently and apart from the M2M single fading channels, which are also known as the single fading double-Doppler channels, attention has been given to the so-called cascaded or double multipath fading models. Indeed, these fading models, where the overall channel gain between the transmitter and receiver is obtained as the product of the gains of two statistically independent fading distributions, have been shown to be more realistic and appropriate in the description of the direct M2M communications channels, where the transmitter and receiver are separated by a large distance [Salo 2006b].

Given the importance of double scattering channel distributions in the modeling of M2M communications, it is a must, therefore, to study and investigate their main statistical properties, e.g. PDF, CDF, LCR, ADF,.. As is known, the CDF (or equivalently the outage probability), LCR (or equivalently the frequency of outages), and ADF, which is also known as the average outage duration, represent the important commonly used performance measures in the design and performance evaluation of wireless communications systems. In this context, there are only some few studies available in the literature so far, which are devoted to the statistical characterization of double fading channels. For instance, the statistical properties of the double Rayleigh fading channels have been studied in [Patel 2006, Kovacs 2002a]. Recently, in [Talha 2007], analytical expressions for the main statistical properties like the mean value, variance, PDF, LCR, and ADF of double Rice fading channels have been derived. More recently, Zlatanov et al. [Zlatanov 2008] derived expressions for the LCR and ADF of double Nakagami-$m$ fading channels. Theoretical results for the general case of multihop Rayleigh and Nakagami fading channels, resulting from multiple scattering propagation scenarios, have been reported in [Karagiannidis 2007, Hadzi-Velkov 2008].

Besides, the importance of the statistical characterization of fading channels, the investigation of the statistical properties of the capacity of time-varying fading channels has been a subject of intensive research in the recent years. This is mainly due to the potential application
of the capacity statistics in the design and optimization of the modern wireless communication systems such as those using multiple antennas, adaptive transmission, and user scheduling. In addition to the mean capacity, considerable attention has been devoted to the study of both the first and second order statistics. Particularly, the second order statistics in the form of the ACF, LCR, and ADF provide a deep insight into the dynamical behavior of the capacity encounters in systems with high mobility applications notably in the case of the inter-vehicular communications. For instance, the analysis of the outage capacity of double Rayleigh fading channels has been presented in [Almers 2006, Gesbert 2002]. Recently, Rafiq et al. [Rafiq 2009] derived expressions for the PDF, CDF, LCR, and ADF of the capacity of double Rice and double Rayleigh fading channels.

As it has been mentioned, in addition to all the underlying classical fading channel models, the Hoyt model can also be useful for the description of realistic multipath propagation scenarios [Nakagami 1960, Youssef 2005b]. In the case of a double fading channel, the corresponding multipath propagation effects can be modeled by a double Hoyt process. The objective of this chapter is to investigate the main statistics of the double Hoyt fading channels together with that of the corresponding channel capacity process. In our analysis, the underlying two single Hoyt fading processes, leading to the double Hoyt process, are considered as statistically independent, but not necessarily identically distributed. In this context, we provide theoretical expressions for the mean value, variance, PDF, LCR, and ADF of the double Hoyt fading processes. In addition, an expression for the PDF of the phase of the double Hoyt channel is also derived. Moreover, expressions for the PDF, CDF, LCR, and ADF of the capacity of double Hoyt fading channels are presented. All the derived quantities include the corresponding known results for the double Rayleigh fading channel as a special case. In addition, the double fading channels described by the combinations Rayleigh×Hoyt, Rayleigh×one-sided Gaussian, and double one-sided Gaussian are special cases of the double Hoyt distribution and, therefore, the results obtained, in this work, are all valid for these multipath fading channel models. Furthermore, the validity of the obtained results is demonstrated by computer simulations.

The rest of the chapter is structured as follows. Section 6.1 presents preliminaries and the formulation of the double Hoyt fading channel. Expressions for the main statistical properties of the double fading channel are presented in Section 6.2. Namely, analytical expressions for the mean value, variance, envelope PDF, phase PDF, LCR, and ADF of the corresponding channel model are derived in this Section. The first and second order statistics in terms of the PDF, CDF, LCR, and ADF of the channel capacity are investigated in Section 6.3. Numerical and simulation results of the PDFs, LCR, and ADF expressions for the double Hoyt fading
6.1. The double Hoyt multipath fading model

In Figure 6.1, we show a two-hop AAF communication scenario illustrating the link between the MS\(_T\) and the MS\(_R\) via the fixed RS. Both signals on the uplink and the downlink propagation paths are assumed to undergo independent, but not necessarily identically distributed flat Hoyt fading distortions. In the equivalent complex baseband representation, the overall complex channel gain is given by

\[ \Upsilon(t) = \mu_1(t)\mu_2(t) \]  

(6.1)

where \( \mu_1(t) \) is the uplink channel gain from the MS\(_T\) to the RS, while \( \mu_2(t) \) models the downlink fading channel from the RS to the MS\(_R\). As it has been reviewed in chapter 2, for a Hoyt propagation environment, \( \mu_i(t) \) (\( i = 1, 2 \)) is a zero-mean complex Gaussian process expressed as

\[ \mu_i(t) = \mu_{i1}(t) + j\mu_{i2}(t) \]  

(6.2)

where the real-valued Gaussian processes \( \mu_{i1}(t) \) and \( \mu_{i2}(t) \) have different variances \( \sigma^2_{i1} \) and \( \sigma^2_{i2} \), respectively. In addition, the complex channel gain \( \mu_i(t) \) can also be written as

\[ \mu_i(t) = R_i(t) \exp[j\vartheta_i(t)] \]  

(6.3)

where the amplitude \( R_i(t) = \sqrt{\mu^2_{i1}(t) + \mu^2_{i2}(t)} \) denotes the Hoyt fading process and \( \vartheta_i(t) = \tan^{-1}(\mu_{i2}(t)/\mu_{i1}(t)) \) represents the channel phase. Expressions for the PDF \( p_{R_i}(z) \) of the process \( R_i(t) \) and \( p_{\vartheta_i}(\theta) \) of the phase process \( \vartheta_i(t) \) are, respectively, given in (2.5) and (2.12). Now,
substituting (6.3) in (6.1), the complex process \( \Upsilon(t) \) can be expressed as a function of the processes \( R_1(t), R_2(t), \vartheta_1(t), \) and \( \vartheta_2(t) \) according to
\[
\Upsilon(t) = \Xi(t) \exp [j\Theta(t)] \quad (6.4)
\]
where \( \Xi(t) = |\Upsilon(t)| = R_1(t)R_2(t) \) denotes the double Hoyt process, and \( \Theta(t) = \vartheta_1(t) + \vartheta_2(t) \) represents the phase process of the double Hoyt fading channel. The aim of the following is to investigate the main statistical properties of the double Hoyt process \( \Xi(t) \) and the phase process \( \Theta(t) \).

6.2 Statistical properties of the fading channel

6.2.1 Mean Value and variance

In this section, the mean value and variance of the double Hoyt process \( \Xi(t) \) are derived. Since the Hoyt fading processes \( R_1(t) \) and \( R_2(t) \) are considered to be statistically independent, these statistical quantities can be obtained without the need to resort to the joint PDF of \( R_1(t) \) and \( R_2(t) \). Accordingly, the mean value \( E\{\Xi(t)\} \) of the double Hoyt process \( \Xi(t) \) can be expressed as
\[
E\{\Xi(t)\} = E\{R_1(t)\} E\{R_2(t)\}. \quad (6.5)
\]
An expression for the mean value \( E\{R_i(t)\} \) \( (i = 1, 2) \) of the Hoyt fading process \( R_i(t) \) can be determined according to [Papoulis 2002]
\[
E\{R_i(t)\} = \int_0^\infty z p_{R_i}(z) \, dz. \quad (6.6)
\]
By substituting (2.5) in (6.6) and using [Gradshteyn 1994, Eqs. 6.621(1) and 8.406(3)], an expression for \( E\{R_i(t)\} \) can be obtained as
\[
E\{R_i(t)\} = \frac{2\sqrt{\pi}}{A_{1i}} \sigma_i F\left(\frac{3}{4}, \frac{5}{4}; 1; \frac{A_{2i}^2}{4\sigma_i^2}\right), \quad i = 1, 2 \quad (6.7)
\]
where \( \sigma_i^2 = \sigma_{\vartheta_1}^2 + \sigma_{\vartheta_2}^2 \), \( F(\cdot) \) denotes the hypergeometric function [Gradshteyn 1994, Eq. (9.100)], and the quantities \( A_{1i} \) and \( A_{2i} \) are given by
\[
A_{1i} = \left((1 + q_i^2)/q_i\right)^2, \quad A_{2i} = \left(1 - q_i^4\right)/q_i^2. \quad (6.8)
\]
6.2. Statistical properties of the fading channel

In (6.8), \( q_i \) \((i = 1, 2)\) stands for the Hoyt fading parameter defined by

\[
q_i = \frac{\sigma_{i2}}{\sigma_{i1}} \quad \text{with} \quad 0 \leq q_i \leq 1. \tag{6.9}
\]

Then, substituting (6.7) in (6.5) results in the following expression for the mean value of the double Hoyt process \( \Xi(t) \)

\[
\mathbb{E}\{\Xi(t)\} = \frac{4\pi\sigma_1\sigma_2}{A_{11}A_{12}} F\left(\frac{3}{4}, \frac{5}{4}; \frac{A_{21}^2}{A_{11}^2} \right) F\left(\frac{3}{4}, \frac{5}{4}; \frac{A_{22}^2}{A_{12}^2} \right). \tag{6.10}
\]

It should be noted that for the special case when \( q_1 = q_2 = 1 \), i.e., the double Rayleigh fading case, the calculation of the above quantity yields the mean value of the double Rayleigh process given by \( \mathbb{E}\{\Xi(t)\} = (\pi/2)\sigma_{11}^2 \) [Kovacs 2002a]. According to [Papoulis 2002], the variance \( \text{Var}\{\Xi(t)\} \) of the process \( \Xi(t) \) can be determined from

\[
\text{Var}\{\Xi(t)\} = \mathbb{E}\{\Xi^2(t)\} - \mathbb{E}\{\Xi(t)\}^2 \tag{6.11}
\]

where \( \mathbb{E}\{\Xi^2(t)\} \) is the mean power of \( \Xi(t) \). Again, based on the assumption of the statistical independence of the processes \( R_1(t) \) and \( R_2(t) \), \( \mathbb{E}\{\Xi^2(t)\} \) can be written as

\[
\mathbb{E}\{\Xi^2(t)\} = \mathbb{E}\{R_1^2(t)\} \mathbb{E}\{R_2^2(t)\} \tag{6.12}
\]

Since the mean power of the Hoyt fading process \( R_i(t) \) is easily determined to be \( \mathbb{E}\{R_i^2(t)\} = \sigma_i^2 \), the corresponding mean power of the double Hoyt process \( \Xi(t) \) is deduced to be

\[
\mathbb{E}\{\Xi^2(t)\} = \sigma_1^2\sigma_2^2. \tag{6.13}
\]

Now, the substitution of (6.10) and (6.13) in (6.11) yields the following expression for the variance \( \text{Var}\{\Xi(t)\} \) of the double Hoyt fading process \( \Xi(t) \)

\[
\text{Var}\{\Xi(t)\} = \sigma_1^2\sigma_2^2 \left\{ 1 - \frac{4\pi}{A_{11}A_{12}} F\left(\frac{3}{4}, \frac{5}{4}; \frac{A_{21}^2}{A_{11}^2} \right) F\left(\frac{3}{4}, \frac{5}{4}; \frac{A_{22}^2}{A_{12}^2} \right) \right\}^2 \tag{6.14}
\]

Again, it should be mentioned that for the special case corresponding to \( q_1 = q_2 = 1 \), the expression in (6.14) results in the variance of the double Rayleigh fading channel [Kovacs 2002a].
6.2.2 PDF of the envelope and phase processes

In this section, we present the PDF of the double Hoyt process $\Xi(t)$ and that of the corresponding phase process $\Theta(t)$. Following [Papoulis 2002], the PDF of $\Xi(t)$ can be obtained using

$$p_{\Xi}(z) = \int_{-\infty}^{\infty} \frac{1}{|y|} p_{R_1R_2} \left( \frac{z}{y}, y \right) dy$$

(6.15)

where $p_{R_1R_2}(x, y)$ is the joint PDF of the envelopes $R_1(t)$ and $R_2(t)$. Due to the statistical independence assumption imposed on the processes $R_1(t)$ and $R_2(t)$, this joint PDF can be written as the product of the two marginal PDFs $p_{R_1}(x)$ and $p_{R_2}(y)$, i.e., $p_{R_1R_2}(x, y) = p_{R_1}(x)p_{R_2}(y)$. Hence, the PDF of the double Hoyt process $\Xi(t)$ is given by

$$p_{\Xi}(z) = \frac{\sqrt{A_{11}A_{12}}}{\sigma_1^2 \sigma_2^2} \int_{0}^{\infty} \frac{1}{y} \exp \left[ -\frac{A_{11}z^2}{4\sigma_1^2 y^2} \right] \exp \left[ -\frac{A_{12}}{4\sigma_2^2 y^2} \right] I_0 \left( \frac{A_{21}}{4\sigma_1^2 y^2} \right) I_0 \left( \frac{A_{22}}{4\sigma_2^2 y^2} \right) dy. \quad (6.16)$$

Then, the integral in (6.16) must be solved numerically. For the special case corresponding to the double Rayleigh fading channel, i.e., $q_1 = q_2 = 1$, it is found that the integral can be solved analytically and results in the double Rayleigh distribution, which is known from the study in [Kovacs 2002a] and is expressed in (2.39). Similarly, for the special case corresponding to the double one-sided Gaussian fading channel, i.e., $q_1 \to 0$ and $q_2 \to 0$, (6.16) simplifies to

$$p_{\Xi}(z) = \frac{2}{\pi \sigma_1 \sigma_2} K_0 \left( \frac{z}{\sigma_1 \sigma_2} \right). \quad (6.17)$$

In addition, setting $q_1 = 1$ and $q_2 \to 0$ in (6.16) and using [Gradshteyn 1994, Eq. 3.472(3)], then (6.16) becomes

$$p_{\Xi}(z) = \frac{1}{\sigma_1 \sigma_2} \exp \left( -\frac{z}{\sigma_1 \sigma_2} \right) \quad (6.18)$$

which represents the PDF of the double fading process $\Xi(t)$ corresponding to the Rayleigh×one-sided Gaussian channel. For completeness, we should add that the PDF described by (6.16) is valid for the general case where the cascaded single Hoyt fading channels are independent but not necessarily identically distributed.

The PDF $p_{\Theta}(\theta)$ of the phase process $\Theta(t)$ can be obtained by solving the following integral [Papoulis 2002]

$$p_{\Theta}(\theta) = \int_{-\infty}^{\infty} p_{\Theta_1, \Theta_2}(\theta - \varphi, \varphi) d\varphi \quad (6.19)$$
where \( p_{\theta_1,\theta_2}(\theta_1, \theta_2) \) is the joint PDF of the phase processes \( \vartheta_1(t) \) and \( \vartheta_2(t) \). Since \( \mu_1(t) \) and \( \mu_2(t) \) are statistically independent, the phase processes \( \vartheta_1(t) \) and \( \vartheta_2(t) \) are also statistically independent. Therefore, \( p_{\theta_1,\theta_2}(\theta_1, \theta_2) \) can be written as \( p_{\theta_1,\theta_2}(\theta_1, \theta_2) = p_{\theta_1}(\theta_1) \cdot p_{\theta_2}(\theta_2) \). Using this result and substituting (2.12) in (6.19), the PDF \( p_{\Theta}(\theta) \) of the phase process \( \Theta(t) \) for the double Hoyt channels is obtained to be

\[
p_{\Theta}(\theta) = \frac{q_1 q_2}{(2\pi)^2} \int_{-\pi}^{\pi} \frac{1}{[q_1^2 \cos^2(\theta - \varphi) + \sin^2(\theta - \varphi)] [q_2^2 \cos^2(\varphi) + \sin^2(\varphi)]} d\varphi, \quad -\pi \leq \theta < \pi.
\]

(6.20)

It can easily be shown that for the special case given by \( q_1 = q_2 = 1 \), i.e., the double Rayleigh channel case, the calculation of the above integral yields the PDF of the double Rayleigh model, i.e., \( p_{\Theta}(\theta) = 1/(2\pi), -\pi \leq \theta < \pi \) [Kovacs 2002a].

### 6.2.3 The LCR and ADF of the fading process

The second order statistics of the double Hoyt fading process \( \Xi(t) \), especially the LCR and the ADF quantities, are presented in this section. The LCR describes how often the process \( \Xi(t) \) crosses a given level \( r \) from up to down (or from down to up) per time unit. This quantity is denoted here by \( N_\Xi(r) \) and can be obtained by solving the following integral [Rice 1944, Rice 1945]

\[
N_\Xi(r) = \int_{0}^{\infty} \hat{z} p_{\Xi}(r, \hat{z}) d\hat{z}
\]

(6.21)

where \( p_{\Xi}(z, \hat{z}) \) represents the joint PDF of the double Hoyt process \( \Xi(t) \) and its time derivative \( \hat{\Xi}(t) \) at the same time instant. Following [Pätzold 2002], this joint PDF can be computed by using

\[
p_{\Xi}(z, \hat{z}) = \int_{0}^{\infty} \int_{-\infty}^{\infty} \frac{1}{y^2} p_{R_1 R_2}(z, y) \exp \left[ \frac{-\sqrt{A^{1}_{11}}}{2\sigma^2_1} x^2 \left( q_1^2 \cos^2(\theta) + \sin^2(\theta) \right) \right] \exp \left[ -\frac{\hat{z}^2}{2 (\beta_2 + (\beta_1 - \beta_2) \cos^2(\theta))} \right] \frac{1}{\sqrt{(\beta_2 + (\beta_1 - \beta_2) \cos^2(\theta))}} d\theta.
\]

(6.22)
6.2. Statistical properties of the fading channel

In (6.23), the quantities $\beta_1$ and $\beta_2$ represent the negative curvature of the autocorrelation function $\Gamma_{\mu_1\mu_1}(\tau)$ and $\Gamma_{\mu_2\mu_2}(\tau)$ of the processes $\mu_1(t)$ and $\mu_2(t)$, respectively, at $\tau = 0$, i.e., $\beta_1 = -\Gamma_{\mu_1\mu_1}(0)$ and $\beta_2 = -\Gamma_{\mu_2\mu_2}(0)$. Now, by substituting (6.23) in (6.22) and with the help of [Gradshteyn 1994, Eq. 3.323(27)], we get the following expression for the joint PDF $p_{\Xi\bar{\Xi}}(z, \bar{z})$ of the processes $\Xi(t)$ and $\bar{\Xi}(t)$

\[
\begin{align*}
p_{\Xi\bar{\Xi}}(z, \bar{z}) = & \frac{\sqrt{A_{11}A_{12}}}{(2\pi)^2 \sqrt{2\pi \sigma_1^2 \sigma_2^2}} z \int_0^{2\pi} \int_0^{2\pi} \int_0^\infty \int_0^\infty \frac{1}{\sqrt{z^2D_2 + y^2E_2}} \exp \left[ -\frac{A_{11}}{2q_1^2 \sigma_1^2 y^2 F_2} \right] \\
& \times \exp \left[ -\frac{A_{12}}{2q_2^2 \sigma_2^2 y^2 G_2} \right] \exp \left[ -\frac{(\bar{z}y)^2}{2(z^2D_2 + y^2E_2)} \right] d\theta_1 d\theta_2 dy \tag{6.24}
\end{align*}
\]

where the quantities $D_2$, $E_2$, $F_2$, and $G_2$ are given by

\[
\begin{align*}
D_2 &= (\beta_{22} + (\beta_{21} - \beta_{22}) \cos^2(\theta_2)) \\
E_2 &= (\beta_{12} + (\beta_{11} - \beta_{12}) \cos^2(\theta_1)) \\
F_2 &= (q_1^2 \cos^2(\theta_1) + \sin^2(\theta_1)) \\
G_2 &= (q_2^2 \cos^2(\theta_2) + \sin^2(\theta_2)).
\end{align*}
\]

Finally, by substituting (6.24) in (6.21) and performing again some algebraic manipulations, the LCR $N_\Xi(r)$ of the double Hoyt fading process $\Xi(t)$ can be expressed in the final form as

\[
N_\Xi(r) = \frac{\sqrt{A_{11}A_{12}}}{(2\pi)^2 \sqrt{2\pi \sigma_1^2 \sigma_2^2}} r \int_0^{2\pi} \int_0^{2\pi} \int_0^\infty \int_0^\infty \frac{1}{\sqrt{r^2D_2 + y^2E_2}} \frac{r^2}{D_2 + y^2E_2} \\
\times \exp \left[ -\frac{A_{11}}{2q_1^2 \sigma_1^2 y^2 F_2} \right] \exp \left[ -\frac{A_{12}}{2q_2^2 \sigma_2^2 y^2 G_2} \right] d\theta_1 d\theta_2 dy. \tag{6.26}
\]

Again, by letting $q_1 = q_2 = 1$, the obtained quantity (6.26) reduces to the LCR of the double Rayleigh process reported in [Patel 2006]. Here, it should be emphasized that $N_\Xi(r)$ is useful for studying the frequency of outages, which is a performance measure commonly used in the wireless communications.

In addition to the LCR, the ADF quantity is of a great importance to the characterization of the fading channels. The ADF of the double Hoyt process $\Xi(t)$ is denoted here by $T_\Xi\bar{\Xi}(r)$. This quantity represents the expected value for the length of the time intervals during which the
6.3. Statistical properties of the channel capacity

The normalized time varying capacity process, which is also known as the instantaneous spectral efficiency, of the double Hoyt fading channels can be expressed using Shannon’s universal equation [Foschini 1998] as

\[ C(t) = \log_2 \left( 1 + \tilde{\gamma} \Xi^2(t) \right) \]  

(6.29)

where the process \( \Xi^2(t) \) represents the channel power gain and the quantity \( \tilde{\gamma} \) denotes the average SNR. In the following, the statistical properties of the capacity of narrowband double Hoyt fading channels are studied. Toward this end, analytical expressions for the PDF and CDF of the instantaneous channel capacity process are derived. Furthermore, for the characterization of the dynamical behavior of the time-varying channel capacity, expressions are provided for the LCR and the ADF. Since the double Rayleigh fading channel is a special case of the double Hoyt model, it is shown that the derived expressions can be reduced to the corresponding results already known for the capacity of the double Rayleigh fading channel.
6.3.1 PDF and CDF of the channel capacity

It can be noted from (6.29) that the capacity $C(t)$ appears as a mapping of the random process $\Xi(t)$. Thus, analytical expressions for the statistics of the capacity $C(t)$ can, in general, be derived from those of $\Xi(t)$ reported in previous parts. Specifically, the PDF of the capacity $C(t)$ of double Hoyt fading channels can be obtained by applying the concept of transformation of random variables [Papoulis 2002, p. 130]. This yields, by using $z = (2^c - 1)/\bar{\gamma}$, the following relation

$$p_C(c) = \frac{2^c \ln(2)}{\bar{\gamma}} p_{\Xi}(\frac{2^c - 1}{\bar{\gamma}}). \quad (6.30)$$

For the determination of the PDF $p_{\Xi}(z)$ of the process $\Xi(t)$, we again apply the concept of transformation of random variables. As a result, this PDF can be expressed as

$$p_{\Xi}(z) = \frac{1}{2\sqrt{z}} p_{\Xi}(\sqrt{z}). \quad (6.31)$$

On the assumption that the Hoyt processes $R_1(t)$ and $R_2(t)$ are statistically independent, we first substitute (6.16) in (6.31) and then (6.31) in (6.30). Finally, the PDF $p_C(c)$ of the instantaneous capacity $C(t)$ of double Hoyt channels is found to be

$$p_C(c) = \frac{2^c \ln(2) \sqrt{A_{11}A_{12}}}{2\bar{\gamma} \sigma_1^2 \sigma_2^2} \int_0^\infty \frac{1}{y} \exp \left( - \frac{A_{11} 2^c - 1}{4\sigma_1^2 \bar{\gamma}} \right) \exp \left( - \frac{A_{12} y^2}{4\sigma_2^2} \right) dy \times I_0 \left[ \frac{A_{11} 2^c - 1}{4\sigma_1^2 \bar{\gamma}} y^2 \right] I_0 \left[ \frac{A_{12} y^2}{4\sigma_2^2} \right] dy. \quad (6.32)$$

The integral involved in (6.32) has to be evaluated by using numerical integration techniques. It should be mentioned that for the special case $q_1 = q_2 = 1$ corresponding to the double Rayleigh fading channel, (6.32) simplifies to the known result in [Rafiq 2009]

$$p_C(c) = \frac{2^c \ln(2)}{2\sigma_{11}^2 \bar{\gamma}} \int_0^\infty \frac{1}{y} \exp \left( - \frac{1}{2\sigma_{11}^2 \bar{\gamma}} 2^c - 1 \right) \exp \left( - \frac{1}{2\sigma_{11}^2 \bar{\gamma}} y^2 \right) dy. \quad (6.33)$$

The CDF $F_C(c)$ of the capacity $C(t)$ of double Hoyt channels can be obtained according to

$$F_C(c) = P_{C-}(c) = \int_0^c p_C(z) dz. \quad (6.34)$$
6.3. Statistical properties of the channel capacity

Substituting (6.32) in (6.34), allows us to obtain the following expression for \( F_C(c) \)

\[
F_C(c) = \frac{\sqrt{A_{11}A_{12}\ln(2)}}{2\sigma_1^2\sigma_2^2\bar{\gamma}} \int_0^c \int_0^\infty \frac{2z}{y} \exp \left[ -\frac{A_{11}}{4\sigma_1^2\bar{\gamma}} \frac{2z - 1}{y^2} \right] \\
\times \exp \left[ -\frac{A_{12}}{4\sigma_2^2y^2} \right] I_0 \left[ \frac{A_{21}}{4\sigma_1^2\bar{\gamma}} \frac{2z - 1}{y^2} \right] I_0 \left[ \frac{A_{22}}{4\sigma_2^2y^2} \right] \, dy \, dz.
\]

(6.35)

The integrals in (6.35) can be solved numerically. Again, by setting \( q_1 = q_2 = 1 \), the expression in (6.35) results in the CDF of the capacity of the double Rayleigh channels given by [Rafiq 2009, Eq. (18)]. Here, it should be emphasized that \( F_C(c) \) is useful for studying the outage probability, which is a performance measure commonly used in wireless communications.

6.3.2 LCR and ADF of the channel capacity

The PDF and CDF derived above do not give any indication concerning the effect of the channel Doppler spread on the behavior of the instantaneous capacity process. Generally, the study of this effect requires the knowledge of the LCR and ADF. In this section, we determine expressions for the LCR and ADF of the capacity \( C(t) \) of the double Hoyt channels. Like the LCR \( N_\Xi(r) \) of double Hoyt fading, the LCR \( N_C(c) \) of the corresponding channel capacity describes the average number of up-crossings (or down-crossings) of the capacity through a fixed level within a time interval of one second. This LCR is defined by [Rice 1944, Rice 1945]

\[
N_C(c) = \int_0^\infty \dot{c} p_{CC} (c, \dot{c}) \, dc
\]

(6.36)

where \( p_{CC} (c, \dot{c}) \) represents the joint PDF of the processes \( C(t) \) and its time derivative \( \dot{C}(t) \). Thus, in order to find the LCR of the capacity of double Hoyt fading channels, the expression for this joint PDF is required. From the transformation of random variables using \( z = \frac{(2^c - 1)}{\bar{\gamma}}, \, \dot{z} = \frac{(2^c \ln(2))}{\bar{\gamma}} \), and with the corresponding Jacobian determinant \( J = \frac{(2^c \ln(2))}{\bar{\gamma}} \), the joint PDF \( p_{CC} (c, \dot{c}) \) can be written as

\[
p_{CC} (c, \dot{c}) = \left( \frac{2^c \ln(2)}{\bar{\gamma}} \right)^2 p_{\Xi^2\dot{\Xi}^2} \left( \frac{2^c - 1}{\bar{\gamma}}, \frac{2^c \ln(2)}{\bar{\gamma}} \right)
\]

(6.37)

where \( p_{\Xi^2\dot{\Xi}^2} (z, \dot{z}) \) stands for the joint PDF of the processes \( \Xi^2(t) \) and its time derivative \( \dot{\Xi}^2(t) \). Based upon the transformation of random variables \( x = \sqrt{z}, \, \dot{x} = \dot{z}/(2\sqrt{z}) \), and with the corresponding Jacobian determinant given by \( J = 1/(4z) \), this joint PDF can be expressed as a
6.3. Statistical properties of the channel capacity

function of the joint PDF \( p_{zz}(z, \dot{z}) \) of the processes \( \Xi(t) \) and \( \dot{\Xi}(t) \) according to

\[
p_{zz}(z, \dot{z}) = \frac{1}{4z} p_{zzz} \left( \sqrt{z}, \frac{\dot{z}}{2\sqrt{z}} \right). \tag{6.38}
\]

Now, substituting the joint PDF \( p_{zz}(z, \dot{z}) \), as presented in (6.24), in (6.38) and carrying out some algebraic manipulations, the joint PDF \( p_{zzzz}(z, \dot{z}) \) of the process \( \Xi(t) \) and its time derivative \( \dot{\Xi}(t) \) is found to be

\[
p_{zzzz}(z, \dot{z}) = \frac{\sqrt{A_{11}A_{12}}}{(4\pi)^2 \sqrt{2\pi\sigma_1^2 \sigma_2^2 \gamma}} \int_0^{2\pi} \int_0^{\infty} \frac{1}{\sqrt{zD_2 + y^4E_2}} \exp \left[ -\frac{\sqrt{A_{11}}}{2q_1\sigma_1^2} \frac{z}{y^2} F_2 \right] \\
\times \exp \left[ -\frac{\sqrt{A_{12}}}{2q_2\sigma_2^2} y^2 G_2 \right] \exp \left[ -\frac{(\dot{z}y)^2}{8z[zD_2 + y^4E_2]} \right] d\theta_1 d\theta_2 dy. \tag{6.39}
\]

Then, by substituting (6.39) in (6.37), we obtain the following expression for the joint PDF \( p_{\dot{z}z}(c, \dot{c}) \)

\[
p_{\dot{z}z}(c, \dot{c}) = \frac{\sqrt{A_{11}A_{12}}}{(4\pi)^2 \sqrt{2\pi\sigma_1^2 \sigma_2^2 \gamma}} \int_0^{2\pi} \int_0^{\infty} \frac{1}{\sqrt{\left(\frac{2c - 1}{\gamma}\right)D_2 + y^4E_2}} \\
\times \exp \left[ -\frac{(2c\ln(2) y)^2}{8(2c - 1)[(2c - 1)D_2 + y^4E_2]} \right] \exp \left[ -\frac{\sqrt{A_{11}}}{2q_1\sigma_1^2} \frac{(2c - 1)}{\gamma} F_2 \right] \\
\times \exp \left[ -\frac{\sqrt{A_{12}}}{2q_2\sigma_2^2} y^2 G_2 \right] d\theta_1 d\theta_2 dy. \tag{6.40}
\]

Finally, after substituting (6.40) into (6.36) and carrying out some algebraic manipulations, we get the desired LCR \( N_C(c) \) as

\[
N_C(c) = \frac{\sqrt{A_{11}A_{12}}}{(2\pi)^{5/2} \sigma_1^2 \sigma_2^2} \int_0^{2\pi} \int_0^{\infty} \frac{1}{\sqrt{\frac{2c - 1}{\gamma}D_2 + E_2}} \\
\times \exp \left[ -\frac{\sqrt{A_{11}}}{2q_1\sigma_1^2 y^2} \frac{(2c - 1)}{\gamma} F_2 \right] \exp \left[ -\frac{\sqrt{A_{12}}}{2q_2\sigma_2^2} y^2 G_2 \right] d\theta_1 d\theta_2 dy. \tag{6.41}
\]

The integrals in (6.41) have to be solved numerically. It should be noted that for the double Rayleigh fading channel, i.e., \( q_1 = q_2 = 1 \), it is found that the threefold integral in (6.41) can be reduced to a single integral and results in the LCR of the double Rayleigh fading channels,
which is known from the study in [Rafiq 2009], as

\[ N_C(c) = \frac{\sqrt{2^c - 1}}{\sqrt{2\pi \gamma \sigma_{11}}} \int_0^\infty \sqrt{\beta_{12} + \beta_{11}} \left( \frac{2^c - 1}{\gamma y^4} \right) \exp \left[ -\frac{1}{2\sigma_{11}^2} \left( \frac{2^c - 1}{\gamma} \right) \right] \exp \left[ -\frac{y^2}{2\sigma_{11}^2} \right] dy. \]  

(6.42)

It is worth mentioning that the derived expression (6.41) for the LCR of the channel capacity could as well be obtained directly from the LCR \( N(\tau) \) of the envelope of double Hoyt fading channels given in (6.26) by just replacing the crossing level \( r \) by its equivalent quantity in terms of the capacity \( \tau = \frac{p(2c_1)}{\gamma} \), i.e.,

\[ N_C(c) = N_\Xi \left( \sqrt{\frac{2^c - 1}{\gamma}} \right). \]  

(6.43)

More much details on the demonstration of this result can be found in the Appendix C.

The ADF \( T_C(c) \) of the instantaneous capacity \( C(t) \) represents the expected value of the time intervals during which the capacity \( C(t) \) is below at a given level \( c \). Thus, using (6.35) and (6.41), the ADF \( T_C(c) \) can be deduced according to [Rice 1944, Rice 1945]

\[ T_C(c) = \frac{F_C(c)}{N_C(c)}. \]  

(6.44)

### 6.4 Numerical and simulation results

In this section, we present computed numerical results for the derived quantities \( p_\Xi(z) \), \( p_\Theta(\theta) \), \( N_\Xi(r) \), and \( T_\Xi(r) \) of the double Hoyt fading channels together with \( p_C(c) \), \( F_C(c) \), \( N_C(c) \), and \( T_C(c) \) of the corresponding channel capacity process \( C(t) \). In addition, we compare all these results with simulations to confirm the correctness of the theory. In order to simulate the statistical properties of the processes \( \Xi(t) \), \( \Theta(t) \), and \( C(t) \), we use the concept of Rice’s sum-of-sinusoids [Rice 1944, Rice 1945]. According to that principle, the statistics of the Gaussian process \( \mu_{ij}(t) \) \((i, j = 1, 2)\) can be approximated by the deterministic process \( \tilde{\mu}_{ij}(t) \) having the form

\[ \tilde{\mu}_{ij}(t) = \sum_{n=1}^{N_{ij}} c_{ij,n} \cos(2\pi f_{ij,n} t + \theta_{ij,n}), \quad i, j = 1, 2 \]  

(6.45)

where \( c_{ij,n}, f_{ij,n}, \) and \( \theta_{ij,n} \) are the gains, discrete Doppler frequencies, and phases, respectively. The quantity \( N_{ij} \) in (6.45) denotes the number of sinusoids used for the generation of the deterministic process \( \tilde{\mu}_{ij}(t) \). In Figure 6.2, we show the general structure of the deterministic simulation process \( \tilde{\mu}_{ij}(t) \) in its continuous-time representation. For the computation of the model parameters, we use the so-called MEDS method [Pätzold 2002, Hajri 2005]. This method
6.4. Numerical and simulation results

identifies the Doppler coefficients $c_{ij,n}(i,j = 1,2)$ with the following quantities

$$c_{ij,n} = \sqrt{\frac{2\sigma_{ij}}{N_{ij}}}$$

(6.46)

and the Doppler frequencies $f_{1j,n}$ and $f_{2j,n}$ ($j = 1,2$) as

$$f_{1j,n} = f_{T,\text{max}} \sin \left[ \frac{\pi}{2N_{1j}} \left( n - \frac{1}{2} \right) \right]$$

(6.47)

$$f_{2j,n} = f_{R,\text{max}} \sin \left[ \frac{\pi}{2N_{2j}} \left( n - \frac{1}{2} \right) \right].$$

(6.48)

Concerning the phases $\theta_{ij,n}$, they are realizations of a random variable that is uniformly distributed in the interval $[0, 2\pi]$.

6.4.1 Statistics of the fading channel

In the double Hoyt fading channel simulator, the following parameters are used: the numbers of sinusoids are $N_{11} = 11$, $N_{12} = 12$, $N_{21} = 13$, and $N_{22} = 14$. The maximum Doppler frequencies of the mobile station transmitter and receiver are taken to be $f_{T,\text{max}} = 20$ Hz and $f_{R,\text{max}} = 30$ Hz, respectively. Concerning the quantities $\beta_{11}$ and $\beta_{12}$ ($i = 1, 2$), they are related according to

$$\beta_{12} = q_i^2 \beta_{11}$$

(6.49)
6.4. Numerical and simulation results

Figure 6.3: The envelope PDF $p_E(z)$ of double Hoyt fading channels.

where in the case of an isotropic scattering [Jakes 1993], $\beta_{11}$ and $\beta_{21}$ are, respectively, given by

$$\beta_{11} = \frac{2}{1 + q_1^2} \left( \sigma_1 \pi f_{T,max} \right)^2$$

(6.50)

and

$$\beta_{21} = \frac{2}{1 + q_2^2} \left( \sigma_2 \pi f_{R,max} \right)^2.$$  

(6.51)

All the plots presented here are depicted for $\sigma_1^2 = \sigma_2^2 = 1$. In Figure 6.3, we show the theoretical PDF $p_E(z)$ of the double Hoyt fading process $\Xi(t)$ along with the corresponding simulated one, for different combinations of the fading severity parameters $q_1$ and $q_2$. Specifically, the PDF of double one-sided Gaussian ($q_1 = q_2 = 0$), Hoyt×one-sided Gaussian ($q_1 = 0.5, q_2 = 0$), Rayleigh×one-sided Gaussian ($q_1 = 1, q_2 = 0$), double Hoyt ($q_1 = 0.5, q_2 = 0.2$), Rayleigh×Hoyt ($q_1 = 1, q_2 = 0.2$), and double Rayleigh ($q_1 = q_2 = 1$) fading channels, are presented in this figure. As can be seen, a reasonable mutual agreement between theory and simulation results is obtained. On the other hand, it can be noted from the plots that a decrease in the fading severity parameters results in an increase in the PDF maxima and a decrease in the PDF spread. Hence, the lowest PDF maxima and the largest PDF spread correspond to that of the envelope PDF of the double Rayleigh fading channel, i.e., when $q_1 = q_2 = 1$. Then, in Figures 6.4–6.7 we consider the case of independent and identically distributed (iid) double Hoyt fading channels.
6.4. Numerical and simulation results

Figure 6.4: The envelope PDF $p_{\Xi}(z)$ of the double Hoyt process $\Xi(t)$ for various values of the Hoyt fading parameters $q_1$ and $q_2$ ($q_1 = q_2$).

Figure 6.5: The phase PDF $p_{\Theta}(\theta)$ of the process $\Theta(t)$ for various values of the Hoyt fading parameters $q_1$ and $q_2$. 

\( \sigma_1^2 = \sigma_2^2 = 1 \)
\( f_{T,\text{max}} = 20\,\text{Hz} \)
\( f_{R,\text{max}} = 30\,\text{Hz} \)
\( (N_{11}, N_{12}) = (11, 12) \)
\( (N_{21}, N_{22}) = (13, 14) \)
6.4. Numerical and simulation results

Figure 6.6: The LCR \( N_\Xi(r) \) of the double Hoyt process \( \Xi(t) \) for various values of the Hoyt fading parameters \( q_1 \) and \( q_2 \).

Figure 6.7: The ADF \( T_\Xi^-(r) \) of the double Hoyt process \( \Xi(t) \) for various values of the Hoyt fading parameters \( q_1 \) and \( q_2 \).
which are obtained by setting $q_1 = q_2$. In this case, Figure 6.4 shows the theoretical PDF $p_\Xi(z)$ of the double Hoyt fading process $\Xi(t)$, along with the corresponding PDF deduced from the simulation of the channel’s envelope, for different values of the Hoyt fading parameters $q_1$ and $q_2$ with $q_1 = q_2$. Again, a reasonable mutual agreement between theory and simulation can be noted from this figure. Furthermore as expected, the envelope PDF of the double Rayleigh fading channel, i.e., when $q_1 = q_2 = 1$, presents the lowest PDF maxima and the largest PDF spread. A comparison between the analytical phase PDF $p_\Theta(\theta)$ and that obtained by simulation is presented in Figure 6.5 for different values of $q_1$ and $q_2$. As can be seen, an excellent fitting between analytical and simulation results is obtained. In Figure 6.6, we compare the analytical LCR $N_\Xi(r)$ of the double Hoyt process $\Xi(t)$ against the corresponding simulation data. First, it can be observed that theory and simulation results are in accordance with each other. Second, it can be seen that the LCR maxima and the LCR spread increase with the decrease and the increase of the Hoyt fading parameters $q_1$ and $q_2$ ($q_1 = q_2$), respectively. Therefore, the LCR with the lowest maxima and the largest spread is obtained for the double Rayleigh fading model, i.e., $q_1 = q_2 = 1$. The excellent fitting between the theoretical and simulation ADF $T_\Xi_-(r)$ of the double Hoyt channel can be studied from Figure 6.7.

6.4.2 Statistics of the channel capacity

For the simulation of the channel capacity, we consider the same specified values for the simulation parameters as mentioned above. In addition, the SNR is set to $\bar{\gamma} = 30$ dB. A comparison between the theoretical PDF $p_C(c)$ of the channel capacity $C(t)$ and the corresponding simulated one is provided in Figure 6.8 for different values of the Hoyt fading parameters $q_1$ and $q_2$. As it can be seen, there is an excellent fitting between the theoretical and simulation results. In addition, this figure shows that the values of $q_1$ and $q_2$ have a significant influence on the PDF of the capacity of the double Hoyt channels. Specifically, with an increase of $q_1$ and $q_2$, the maximum value of $p_C(c)$ increases, while the spread is seen to decrease. Hence, as expected, the highest capacity can be achieved if $q_1 = q_2 = 1$ corresponding to the double Rayleigh fading channels. The analytical CDF $F_C(c)$ of the channel capacity is shown in Figure 6.9 in comparison with that obtained from computer simulations for different values of $q_1$ and $q_2$. Also in this case, an excellent agreement between theory and simulation can be observed. As expected, the outage performance deteriorates with a decrease in the fading severity parameters $q_1$ and $q_2$. In Figure 6.10, we study the influence of the parameters $q_1$ and $q_2$ on the LCR of the channel capacity. Again, the simulation results are seen to be in a very good agreement with the analytical ones. For the range of small values of the capacity level $c$, which is of a
6.4. Numerical and simulation results

Figure 6.8: The PDF $p_C(c)$ of the capacity of double Hoyt fading channels for various values of $q_1$ and $q_2$.

Figure 6.9: The CDF $F_C(c)$ of the capacity of double Hoyt fading channels for various values of $q_1$ and $q_2$. 
6.4. Numerical and simulation results

Figure 6.10: The LCR $N_C(c)$ of the capacity of double Hoyt fading channels for various values of $q_1$ and $q_2$.

Figure 6.11: The ADF $T_C(c)$ of the capacity of double Hoyt fading channels for various values of $q_1$ and $q_2$. 

- Theory
- Simulation

$\sigma^2_1 = \sigma^2_2 = 1$
$\g = 30$ dB
$f_{T,\text{max}} = 20$ Hz
$f_{R,\text{max}} = 30$ Hz
6.5 Conclusions

practical interest, the LCR increases significantly with a decrease of $q_1$ and $q_2$. For high values of $c$, however, the impact of these parameters on the LCR is much less pronounced. Finally, a comparison between the theoretical ADF $T_C(c)$ and that obtained from simulations is provided by Figure 6.11 for different values of $q_1$ and $q_2$. Again, it can be observed that there exists a reasonable correspondence between theory and simulation. For low values of $c$, the ADF decreases as the fading severity parameters are increased, meaning that the performance in terms of the average outage duration improves. For completeness, it is worthwhile to mention that we do not observe any significant change in the behavior of the second order statistics of the capacity processes compared to those of the envelope of the double Hoyt fading channels, which have been reported in the previous sections. This is attributed to the fact that the capacity is a monotonic function of the fading channel power gain.

6.5 Conclusions

In this chapter, a study on the statistical properties of the double Hoyt fading channels and their corresponding channel capacity process has been presented. First, analytical expressions for the mean value, variance, PDF, LCR, and ADF of the double Hoyt process have been derived. Then, an expression for the PDF of the channel phase has been provided. Moreover, analytical expressions for the PDF, CDF, LCR, and ADF of the channel capacity have been derived. All the obtained analytical expressions include known results corresponding to the double Rayleigh fading channel as a special case. Additionally, the validity of the derived expressions has been confirmed by computer simulations. As it has been mentioned, the CDF, LCR, and ADF measures are of a crucial importance in the design and performance evaluation of wireless communication systems.

Besides all these investigated performance measures, the BEP represents the more appropriate performance measure revealing the nature of the system behavior. Accordingly, it is a must and of a great interest to study and analyze the BEP performance of M2M communications over double Hoyt fading channels. In this context and drawing upon the obtained envelope PDF of the double Hoyt fading channels, the next chapter aims at contributing to the topic of performance analysis of various digital modulation schemes over the double Hoyt channels perturbed by AWGN.
The present chapter aims at contributing to the performance analysis of M2M communications over the double Hoyt fading channels, by studying the BEP of several modulation schemes commonly used in wireless communications.

Namely, we investigate here the impact of the frequency-flat double Hoyt propagation environments on the error rate performance of coherent BPSK, QPSK, FSK, MSK, and ASK modulation schemes. Starting from the envelope PDF of the double Hoyt fading channels, which has been derived in chapter 6, we first give an analytical expression for the PDF of the receiver output SNR. Then, using this PDF and drawing upon the theory reported in [Simon 2005, Proakis 2001], an analytical expression, involving a double (definite and semi-infinite) integral, is presented for the BEP of all the modulation techniques mentioned above. The obtained formula is generic and includes, as special cases, the BEP corresponding to the double one-sided Gaussian, Rayleigh×one-sided Gaussian, Rayleigh×Hoyt, Hoyt×one-sided Gaussian, and double Rayleigh fading channel models. For most of these special cases, the BEP formula is shown to simplify to closed-form expressions which involve a single integral with finite limits. The Rayleigh×Hoyt propagation environment can as well be encountered in satellite based mobile communications where the ionospheric scintillation is known to be described by the Hoyt distribution [Chytil 1967], while the terrestrial multipath fading is characterized by the Rayleigh channel model. Numerical examples are presented to analyze the behavior of the BEP for various cases of the fading environments. Moreover, the validity of the analytical results is demonstrated by using computer simulations for the BPSK modulation scheme.

The remainder of the chapter is organized as follows. In Section 7.1, we first present preliminaries and then derive the PDF of the instantaneous SNR. The average BEP of coherent modulation schemes previously mentioned, i.e., BPSK, QPSK, FSK, MSK, and ASK, is investigated in Section 7.2. Numerical results and analysis are provided in Section 7.3, while the conclusion is drawn in Section 7.4.
7.1 The SNR distribution

We consider a radio link between a single-input single-output mobile transmitter and receiver. The propagation conditions between the two mobiles are modeled statistically by a double Hoyt fading distribution. By assuming a frequency non-selective fading, the complex valued equivalent baseband signal, present at the input of the mobile receiver, can be written as \[ y(t) = \Xi(t)x(t)e^{j\Theta(t)} + n(t) \] (7.1)

where \( x(t) \) is the modulated signal and \( n(t) \) represents the complex AWGN with a one-sided power spectral density \( N_0 \). As it has been demonstrated in chapter 6, the PDF of the double Hoyt process \( \Xi(t) \) is expressed by (6.16). This PDF expression can also be written as

\[
p_{\Xi}(z) = \frac{1 + q_1^2}{q_1 q_2 \sigma_1^2 \sigma_2^2} \int_0^\infty \frac{1}{y} \exp \left[ -\frac{(1 + q_1^2)^2 z^2}{4q_1^2 \sigma_1^2 y^2} \right] \exp \left[ -\frac{(1 + q_2^2)^2}{4q_2^2 \sigma_2^2} y \right] dy \times I_0 \left[ \frac{1 - q_1^4}{4q_1^2 \sigma_1^2} \frac{z^2}{y^2} \right] I_0 \left[ \frac{1 - q_2^4}{4q_2^2 \sigma_2^2} \frac{y}{y^2} \right] dy.
\] (7.2)

As it is well known, the determination of the average BEP requires the knowledge of the distribution of the instantaneous SNR per bit. The instantaneous output SNR per bit is defined as

\[
\gamma_s(t) = \Xi^2(t) \frac{E_b}{N_0}
\] (7.3)

where \( E_b \) is the average received bit energy. Now, based on the concept of transformation of random variables [Papoulis 2002, p. 130], the PDF of the instantaneous SNR \( \gamma_s(t) \) is easily obtained from (7.3) combined with (7.2) according to

\[
p_{\gamma_s}(\beta) = \frac{1}{2 \sqrt{\beta (E_b/N_0)}} p_{\Xi} \left( \sqrt{\frac{\beta}{E_b/N_0}} \right).
\] (7.4)

Hence, substituting (7.2) in (7.4) yields the following expression for \( p_{\gamma_s}(\beta) \)

\[
p_{\gamma_s}(\beta) = \frac{1 + q_1^2}{2 q_1 q_2 \gamma_s} \int_0^\infty \frac{1}{y} \exp \left[ -\frac{1 + q_2^2}{4q_2^2 \sigma_{21}^2 y^2} y \right] \exp \left[ -\frac{(1 + q_1^2)^2 (1 + q_2^2) \sigma_{21}^2 \beta}{4q_1^2 y^2} \right] dy \times I_0 \left[ \frac{1 - q_2^2}{4q_2^2 \sigma_{21}^2} \frac{y}{y^2} \right] I_0 \left[ \frac{(1 - q_1^4) (1 + q_2^2) \sigma_{21}^2 \beta}{4q_1^2 y^2} \right] dy
\] (7.5)
where $\bar{\gamma}_s$ is the average SNR given by

$$\bar{\gamma}_s = \sigma_1^2 \sigma_2^2 \frac{E_b}{N_0}. \quad (7.6)$$

The integral involved in (7.5) can then be evaluated by using numerical techniques. However, for the following special cases, the integration can be carried out to yield a closed-form solution for the PDF of $\gamma_s(t)$. Namely, in the case of $q_1 = q_2 = 1$, i.e., the case of the double Rayleigh fading model, the integral can be solved with the help of [Gradshteyn 1994, Eq. 8.432(6.7)]. This yields

$$p_{\gamma_s}(\beta) = \frac{2}{\bar{\gamma}_s} K_0 \left(2\sqrt{\frac{\beta}{\bar{\gamma}_s}}\right) \quad (7.7)$$

Similarly, for the special case corresponding to the double one-sided Gaussian fading channel, i.e., $q_1 \to 0$ and $q_2 \to 0$, (7.5) simplifies to

$$p_{\gamma_s}(\beta) = \frac{1}{\pi \sqrt{3\bar{\gamma}_s}} K_0 \left(\sqrt{\frac{\beta}{\bar{\gamma}_s}}\right). \quad (7.8)$$

In addition, letting $q_1 = 1$ and $q_2 \to 0$ in (7.5) and using [Gradshteyn 1994, Eq. 3.472(3)], then (7.5) becomes

$$p_{\gamma_s}(\beta) = \frac{1}{\sqrt{2\beta\bar{\gamma}_s}} \exp \left(-\sqrt{\frac{2\beta}{\bar{\gamma}_s}}\right) \quad (7.9)$$

which is the PDF of the instantaneous SNR corresponding to the Rayleigh×one-sided Gaussian channel.

For completeness, we should add that the PDF described by (7.5) is valid for the general case where the cascaded single Hoyt fading channels are independent but not necessarily identically distributed. Here, it can be observed from (7.5) that the distribution type remains unchanged regardless of whether the cascaded fading models have identical or non-identical distributions; except that the statistical parameters are different.

### 7.2 BEP derivation

For the derivation of the desired BEP, we apply the approach based on the average calculation of the conditional BEP in an AWGN channel over the instantaneous SNR distribution. That is, the evaluation of the average BEP performance can be accomplished according to [Simon 2005, 104]...
7.2. BEP derivation

Table 7.1: The value of the parameter $p$ for different modulation schemes.

<table>
<thead>
<tr>
<th>Modulation Schemes</th>
<th>$p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coherent binary phase-shift keying (BPSK)</td>
<td>1</td>
</tr>
<tr>
<td>Coherent quadrature phase-shift keying (QPSK)</td>
<td>1</td>
</tr>
<tr>
<td>Coherent frequency-shift keying (FSK)</td>
<td>1/2</td>
</tr>
<tr>
<td>Coherent amplitude-shift keying (ASK)</td>
<td>1/4</td>
</tr>
<tr>
<td>Minimum-shift keying (MSK)</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Proakis 2001]

$$P_b = \int_0^\infty P_b(E|\beta)p_{\gamma_s}(\beta)d\beta$$

(7.10)

where $P_b(E|\beta)$ is the conditional BEP performance in the AWGN channel. For the modulation schemes considered in this work, i.e., BPSK, QPSK, MSK, FSK, and ASK, the conditional BEP is given by [Proakis 2001]

$$P_b(E/\beta) = Q\left(\sqrt{2p\beta}\right)$$

(7.11)

where $Q(\cdot)$ stands for the Gaussian $Q$-function [Craig 1991, Gradshteyn 1994] and $p$ is a parameter that depends on the modulation type as shown in Table 7.1 [Proakis 2001]. For convenience, we use the alternate definite integral form of $Q(\cdot)$ which has been reported in [Craig 1991]

$$Q(x) = \frac{1}{\pi} \int_0^{\pi/2} \exp \left[-\frac{x^2}{2\sin^2\theta}\right] d\theta, \quad x \geq 0.$$

(7.12)

Substituting (7.12) and (7.5) in (7.10), results in the following expression for the average BEP

$$P_b = \frac{(1 + q_1^2)(1 + q_2^2)}{2q_1q_2\gamma_s} \int_0^\infty \frac{1}{y} \exp \left[-\frac{1 + q_2^2}{4q_2^2\sigma_{21}^2} y^2\right] I_0 \left[\frac{1 - q_2^2}{4q_2^2\sigma_{21}^2} y^2\right] J(p, y^2, \gamma_s) dy$$

(7.13)

where

$$J(p, y^2, \gamma_s) = \frac{1}{\pi} \int_0^{\pi/2} d\theta \int_0^\infty \exp \left[-\frac{(1 + q_1^2)^2(1 + q_2^2) \sigma_{21}^2 \beta}{4q_1^2y^2}\right]$$

$$\times I_0 \left[\frac{(1 - q_1^4)(1 + q_2^2) \sigma_{21}^2 \beta}{4q_1^4y^2}\right] \exp \left[-\frac{p\beta}{\sin^2\theta}\right] d\beta.$$  

(7.14)
Now, using [Gradshteyn 1994, Eq. 6.611(4.8)], the semi-infinite range integration in (7.14) can be carried out. This leads to

\[
J(p, y^2; \gamma_s) = \frac{1}{\pi} \int_0^{\pi/2} \frac{\sin^2 \theta}{\sqrt{p + \frac{(1+q_1^2)(1+q_2^2)\sigma_{12}^2}{2q_1y^2\gamma_s}} \sin^2 \theta} d\theta
\]  

(7.15)

Finally, resubstituting (7.15) in (7.13), yields the following solution for the average BEP performance

\[
\mathcal{P}_b = \frac{1 + q_1^2}{2\pi q_1 q_2 \gamma_s} \int_0^{\pi/2} \int_0^{\infty} y \exp \left[ -\frac{1}{4q_2^2 \sigma_{21}^2} y^2 \right] I_0 \left[ \frac{1 - q_2^2}{4q_2^2 \sigma_{21}^2} y^2 \right] \frac{\sin^2 \theta}{\sqrt{p + \frac{(1+q_1^2)(1+q_2^2)\sigma_{12}^2}{2q_1y^2\gamma_s}} \sin^2 \theta} d\theta dy.
\]  

(7.16)

The integrals involved in (7.16) are then computed numerically. In the following, we examine the special cases of the obtained average BEP. For the double Rayleigh fading channel, i.e., \(q_1 = q_2 = 1\), (7.16) can be simplified. Specifically, using [Gradshteyn 1994, Eq. 6.631(3)] and doing some algebraic manipulations, the average BEP \(\mathcal{P}_b\) is reduced to the following exact result

\[
\mathcal{P}_b = \frac{1}{\pi \sqrt{q_1 \gamma_s}} \int_0^{\pi/2} \sin \theta \exp \left[ \frac{\sin^2 \theta}{2q_1 \gamma_s} \right] W_{-1/2, 0} \left( \frac{\sin^2 \theta}{p \gamma_s} \right) d\theta
\]  

(7.17)

where \(W_{-1/2, 0} (\cdot)\) represents the Whittaker function defined by [Gradshteyn 1994, Eq. 9.222(2)]. Similarly, for the special case of the double one-sided Gaussian model, i.e., \(q_1 \to 0\) and \(q_2 \to 0\), based on the use of [Gradshteyn 1994][Eqs. 6.618 (3), 1.411 (9), and (9.72)], (7.16) becomes

\[
\mathcal{P}_b = \frac{1}{2\pi^{3/2} \sqrt{q_1 \gamma_s}} \int_0^{\pi/2} \sin \theta \exp \left[ \frac{\sin^2 \theta}{8q_2 \gamma_s} \right] K_0 \left( \frac{\sin^2 \theta}{8p \gamma_s} \right) d\theta.
\]  

(7.18)

Again, for \(q_1 = 1\) and \(q_2 \to 0\), i.e., the case of the Rayleigh×one-sided Gaussian fading channel, the BEP expression can be simplified by using [Gradshteyn 1994, Eq. 3.322 (2)]. This leads to the result given by

\[
\mathcal{P}_b = \frac{1}{\sqrt{2\pi \gamma_s}} \int_0^{\pi/2} \sin \theta \exp \left[ \frac{\sin^2 \theta}{2p \gamma_s} \right] \text{erfc} \left( \frac{\sin \theta}{\sqrt{2p \gamma_s}} \right) d\theta
\]  

(7.19)
where erfc(·) represents the complementary error function [Gradshteyn 1994]. Concerning the special cases corresponding to Hoyt×Rayleigh and Hoyt×one-sided Gaussian fading models, an analytical solution of the semi-infinite integral in (7.16) is not known. Therefore, the simplified forms of the BEP could not be obtained for these cases. Here, it is worth mentioning that the Hoyt×Rayleigh fading channel is useful in describing realistically the propagation characteristics of the satellite based mobile communications. Indeed, in such communication scenarios, the ionospheric scintillation is described by the Hoyt distribution [Chytil 1967], while the terrestrial multipath fading is characterized by the Rayleigh model. Hence, (7.16) is useful as well in the performance studies of this type of mobile communications.

### 7.3 Numerical and simulation results

In this section, we first verify by the given computer simulations the correctness of the derived expression for the BEP performance. Toward this end, we consider only the case of the coherent BPSK modulation scheme. In order to simulate the double Hoyt fading channel, we employ the concept of Rice’s sum-of-sinusoids [Rice 1945, Rice 1944]. According to that principle, the simulation system of the double Hoyt process ξ(t) has the structure shown in Figure 7.1, where the Gaussian process μ_{ij}(t) (i, j = 1, 2) are approximated by the deterministic process μ_{ij}(t) given in (6.45). The so-called MEDS method is used in the computation of the simulation model parameters. In Figure 7.2, we show the theoretical BEP performance of the coherent BPSK modulation scheme along with the corresponding simulated one, for different combinations of the fading severity parameters q_1 and q_2. Specifically, the performance of the double one-sided Gaussian (q_1 = q_2 = 0), Hoyt×one-sided Gaussian (q_1 = 0.5, q_2 = 0), Rayleigh×one-sided Gaussian (q_1 = 1, q_2 = 0), double Hoyt (q_1 = 0.5, q_2 = 0.2), Rayleigh×Hoyt (q_1 = 1, q_2 = 0.2), and double Rayleigh (q_1 = q_2 = 1) fading channels, are presented in this figure. As it can be seen, a reasonable mutual agreement between theory and simulation results is obtained. In plus, it can be noted from the plots that a decrease in the fading severity parameters results in an increase in the values of the BEP. Hence, as expected, the best case of the BEP performance corresponds to the double Rayleigh fading channels, while the worst BEP performance is obtained in the case of double one-sided Gaussian fading channels. Then, in Figures 7.3–7.5, we plot some numerical examples that illustrate the theoretical BEP performance of various coherent modulation schemes. Figure 7.3 shows the average BEP $P_b$ of coherent BPSK, QPSK, FSK, MSK, and ASK modulations versus the average SNR $\gamma_s$ in the case of Rayleigh×Hoyt fading channels (q_1 = 1, q_2 = 0.2). As it is expected, we can observe from this figure that the best BEP
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Figure 7.1: Structure of the deterministic simulation system for the double Hoyt process $\Xi(t)$.

performance corresponds to the BPSK and QPSK modulation schemes, i.e., the case when the parameter $p = 1$. The average BEPs performance of FSK modulation in Rayleigh × one-sided Gaussian ($q_1 = 1, q_2 = 0$), Rayleigh × Hoyt {($q_1 = 1, q_2 = 0.2$), ($q_1 = 1, q_2 = 0.5$)}, and double Rayleigh ($q_1 = q_2 = 1$) channels are illustrated in Figure 7.4, while those corresponding to the MSK modulation are provided in Figure 7.5. Again, we can notice from these figures that for all the cases, the average BEP performance degrades with decreasing values of $q_2$ given that $q_1 = 1$. Hence, the best BEP performance is obtained for the double Rayleigh fading, when compared to the other considered double fading channels.
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Figure 7.2: The theoretical and simulated BEP performance of a coherent BPSK modulation in the double Hoyt fading channels.

Figure 7.3: The theoretical BEP performance of coherent BPSK, QPSK, FSK, and MSK modulation schemes in Rayleigh×Hoyt fading channel.
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Figure 7.4: The theoretical BEP performance of a coherent FSK modulation scheme in Rayleigh×one-sided Gaussian, Rayleigh×Hoyt, and double Rayleigh fading channels.

Figure 7.5: The theoretical BEP performance of a coherent MSK modulation scheme in Rayleigh×one-sided Gaussian, Rayleigh×Hoyt, and double Rayleigh fading channels.
7.4 Conclusion

In this chapter, we studied the BEP performance of the digital modulated signals that are transmitted over slow and frequency-flat double Hoyt fading channels. Specifically, an expression for the average BEP of coherent BPSK, QPSK, FSK, MSK, and ASK modulation schemes has been derived. The obtained BEP result is general and includes several special cases of the double Hoyt fading channel. Furthermore, the validity of the theoretical expression has been confirmed by computer simulations for the case of BPSK. The results presented in this chapter are useful in the performance assessment of the M2M communication systems operating over propagation environments characterized by fading conditions that are more severe than the double Rayleigh fading. They can also be applied in mobile satellite communications when the overall propagation channel can be statistically described by the Hoyt × Rayleigh fading distribution.
Conclusions and outlook

The performance analysis of wireless communications, taking into account the appropriate propagation characteristics, is essential and highly relevant for designing and optimizing wireless systems. Indeed, this analysis is generally used for verifying whether or not the system under design is capable to meet the specific propagation conditions. Motivated by this fact and given the importance of the M2M communications, which are becoming more and more useful in future wireless systems, the main purpose of this thesis is to provide a performance analysis on wireless M2M communications over the Hoyt fading channels. As is known, the Hoyt fading model is a general multipath fading distribution, which includes the one-sided and Rayleigh fading models as special cases. It also has the advantage of approximating the Nakagami-\(m\) distribution in the range of the fading severity between 0.5 and 1 and vice versa.

Specifically, the objective of this thesis has to contribute to the topic of performance analysis of various digital modulation schemes commonly used in wireless communication systems over the M2M Hoyt fading channels. In this context, this thesis work has been divided into two essential parts. Regarding the first part, which has been exposed in chapters 3, 4, and 5, we have focussed on the topic of the performance analysis of various digital modulation schemes, especially DPSK and FSK with LDI and differential detections, over the M2M single Hoyt fading channels. In this context, significant findings have been obtained. Namely, a formula for the average number of FM clicks occurring at the output of a discriminator FM receiver has been derived. In addition, a closed-form expression for the PDF of the phase difference between two phases or frequency modulated signals transmitted over the Hoyt fading channels and perturbed by the correlated Gaussian noise has been presented. All the above derived formulas have been validated by reducing them to the Rayleigh fading channel as a special case of the Hoyt fading. Furthermore, the presented PDF expression of the overall phase difference has also been checked by comparing it with that obtained from computer simulations, considering a M2M single Hoyt fading channel. Then, based upon this PDF, closed-form expressions for the BEP performance of the above mentioned modulation schemes that are transmitted over the M2M single Hoyt fading channels have been derived and verified to include the special case corresponding to the Rayleigh fading channel. From the obtained BEP results, it has been concluded that the
BEP performance degrades with decreasing values of the Hoyt fading severity parameter $q$ and, therefore, the lowest BEP has been obtained for the Rayleigh fading channel, i.e., $q = 1$, while the highest one corresponds to the one-sided Gaussian channel, i.e., $q = 0$.

Then, in the second part of this thesis, where the corresponding main findings have been presented in chapters 6 and 7, we have introduced the cascaded or double Hoyt fading channels. These scattering fading channels, where the overall complex channel gain between the transmitter and receiver is modeled as the product of the gains of two statistically independent single Hoyt fading channels, can be used in the channel modeling of more realistic and appropriate M2M communication systems operating over propagation environments characterized by fading conditions that are more severe than those described by the double Rayleigh fading. Considering this double scattering channel model, a study on its main statistical properties has been investigated. Namely, analytical expressions for the performance measures commonly used in wireless communications especially, the CDF, LCR, and ADF which are useful for studying the outage probability, frequency of outages, and average outage duration, respectively, have been derived. Furthermore, the first and second order statistics of the capacity of the double Hoyt fading channels have also been studied. In addition to all these investigated performance measures, we have studied in this part, hereafter, the impact of slowly varying frequency flat double Hoyt propagation environments on the error rate performance of coherent BPSK, QPSK, FSK, MSK, and ASK modulation schemes. Hence, a closed-form expression for the average BEP of all these coherent modulation schemes has been investigated. The obtained BEP expression is general and includes, as special cases, results corresponding to the channel combinations given by Rayleigh × Hoyt, Rayleigh × one-sided Gaussian, Hoyt × one-sided Gaussian, double one-sided Gaussian, and double Rayleigh fading channels. Furthermore, the validity of the obtained theoretical results has been checked by means of computer simulations for some of the modulation schemes used in the analysis. It has also been concluded from the presented BEP results, for all considered modulation schemes, that a decrease in the fading severity parameters $q_1$ and $q_2$, which characterize the double Hoyt fading channels, results in an increase in the values of the BEP. Hence, the best case of the BEP performance has corresponded to the double Rayleigh fading channels, i.e., $q_1 = q_2 = 1$, while the worst one has been obtained in the case of double one-sided Gaussian fading channels, i.e., the case when $q_1 = q_2 = 0$.

Let us now close this dissertation with the following key ideas for possible future research.

A significant enhancement, in this work, would be the extension of the presented performance analysis studies on SISO M2M communications over the Hoyt fading channels to the case of
MIMO M2M Hoyt fading channels. As it is known, the employment of multiple antennas at both the transmitter and receiver, i.e., MIMO technologies, is very promising for M2M communications since it can be useful in combating the effects of fading and enables to greatly improve the link reliability and increase the overall system capacity.

Another extension of this work can be done on the second part of the thesis, which deals with the performance analysis of wireless M2M communications over the double Hoyt fading channels. Indeed and in the presented analysis, we have assumed that the channel fading model is slow and non-selective. This assumption would be extended by considering fast varying frequency-selective double Hoyt fading channels, which reflect more realistic propagation environments and high bandwidth transmissions, and try, thereafter, to evaluate the error rate performance of the M2M communications over such fading channels.

Furthermore, the proposed double Hoyt fading model is restricted to a two-single hop. This model can be extended and generalized by the Multihop cascaded Hoyt fading channels, i.e., $N^*Hoyt$. Hence, a study on the statistical characterization of $N^*Hoyt$ fading channels as well as their applications to a related performance analysis of wireless radio links can be investigated.
Appendix A

The BEP parameters for different bit patterns due to ISI effects

This part complements chapter 3. Specifically, the various parameters needed in the calculation of the BEP performance given by (3.39) for the three bit patterns “111”, “010”, and “011” are presented.

- For “111” bit pattern

\[
\Delta \phi = \pi h \\
C_1 = \left| H(h/2T) \right|^2, \quad C_2 = 0 \\
N = 2\pi d \cdot \frac{h}{\sqrt{\left[ \left( |H(h/2T)|^2 + \epsilon \right)^2 - I^2 \right]}}. \tag{A-1}
\]

- For “010” bit pattern

\[
\Delta \phi = 2 \tan^{-1} \frac{n_1}{1 - n_2} \\
C_1 = \left[ \sin \frac{\pi h}{T} \right]^2 \left[ (1 - n_2)^2 + n_1^2 \right], \quad C_2 = 0 \\
\phi(t) = \frac{\pi}{T} \left[ \frac{(n_2 \cos \frac{2\pi t}{T} - 1) n_1 \sin \frac{\pi t}{T} - 2n_1 n_2 \cos \frac{\pi t}{T} \sin \frac{2\pi t}{T}}{(1 - n_2 \cos \frac{2\pi t}{T})^2 + n_1^2 \cos^2 \frac{\pi t}{T}} \right] \\
a^2(t) = \left[ \sin \frac{\pi h}{T} \right]^2 \cdot \left[ \left( 1 - n_2 \cos \frac{2\pi t}{T} \right)^2 + n_1^2 \cos^2 \frac{\pi t}{T} \right]. \tag{A-2}
\]

where the quantities \( n_1 \) and \( n_2 \) are given by

\[
n_1 = 2 |H(1/2T)| \frac{h^2}{1 - h^2} \cot(\pi h/2) \\
n_2 = 2 |H(1/T)| \frac{h^2}{4 - h^2} \tag{A-3}
\]
• For “011” bit pattern

\[
\Delta \phi = \frac{1}{2} [\Delta \phi_{111} + \Delta \phi_{010}]
\]
\[
\mathcal{N} = \frac{1}{2} [\mathcal{N}_{111} + \mathcal{N}_{010}].
\]  
(A-4)
Appendix B

Expressions for the quantities $K$ and $\chi_{ij} \ (i, j = 1, \ldots, 4)$

This Appendix provides, for completeness, the expressions of the quantities $K$ and $\chi_{ij} \ (i, j = 1, \ldots, 4)$, used in chapter 4 for the description of (4.13), (4.16), and (4.20). These quantities are found to be given by

\[
K = 1 - \frac{a_{13}^2}{\sigma_{x_1}^2 \sigma_{x_2}^2} + \frac{a_{24}^2}{\sigma_{y_1}^2 \sigma_{y_2}^2} + \frac{a_{14}^2}{\sigma_{y_1}^2 \sigma_{x_2}^2} + \frac{a_{32}^2}{\sigma_{y_1}^2 \sigma_{x_1}^2} + \frac{a_{34}^2}{\sigma_{x_2}^2 \sigma_{y_2}^2} - \frac{(a_{13}a_{24} - a_{14}a_{32})^2}{\sigma_{y_1}^2 \sigma_{x_2}^2 \sigma_{y_2}^2} - \frac{2a_{34}}{\sigma_{x_2}^2 \sigma_{y_2}^2} \left( \frac{a_{24}a_{32}}{\sigma_{y_1}^2} + \frac{a_{13}a_{14}}{\sigma_{x_1}^2} \right) \quad \text{(B-1)}
\]

and

\[
\chi_{11} = \frac{1}{\sigma_{x_1}^2} \left[ 1 - \frac{a_{13}^2}{\sigma_{x_1}^2 \sigma_{x_2}^2} + \frac{a_{24}^2}{\sigma_{y_1}^2 \sigma_{x_2}^2} + \frac{a_{14}^2}{\sigma_{y_1}^2 \sigma_{x_2}^2} - \frac{2a_{32}a_{34}}{\sigma_{y_1}^2 \sigma_{x_2}^2} \right]
\]

\[
\chi_{22} = \frac{1}{\sigma_{y_1}^2} \left[ 1 - \frac{a_{13}^2}{\sigma_{x_1}^2 \sigma_{y_2}^2} + \frac{a_{14}^2}{\sigma_{x_1}^2 \sigma_{y_2}^2} + \frac{a_{34}^2}{\sigma_{y_1}^2 \sigma_{y_2}^2} - \frac{2a_{32}a_{14}}{\sigma_{y_1}^2 \sigma_{y_2}^2} \right]
\]

\[
\chi_{12} = \frac{1}{\sigma_{x_1}^2 \sigma_{y_1}^2} \left[ \frac{a_{24}a_{14}}{\sigma_{y_2}^2} + \frac{a_{13}a_{32}}{\sigma_{x_2}^2} - \frac{1}{\sigma_{x_2}^2} \left( a_{14}a_{32}a_{34} + a_{13}a_{24}a_{34} \right) \right]
\]

\[
\chi_{33} = \frac{1}{\sigma_{x_2}^2} \left[ 1 - \frac{a_{14}^2}{\sigma_{y_1}^2 \sigma_{x_2}^2} + \frac{a_{32}^2}{\sigma_{y_1}^2 \sigma_{x_2}^2} \right]
\]

\[
\chi_{44} = \frac{1}{\sigma_{y_2}^2} \left[ 1 - \frac{a_{13}^2}{\sigma_{x_1}^2 \sigma_{y_2}^2} + \frac{a_{34}^2}{\sigma_{x_1}^2 \sigma_{y_2}^2} \right]
\]

\[
\chi_{34} = \frac{1}{\sigma_{x_2}^2 \sigma_{y_2}^2} \left[ \frac{a_{24}a_{32}}{\sigma_{y_1}^2} + \frac{a_{14}a_{13}}{\sigma_{x_1}^2} - a_{34} \right]
\]

\[
\chi_{13} = \frac{1}{\sigma_{x_1}^2 \sigma_{x_2}^2} \left[ \frac{a_{13}}{\sigma_{y_1}^2} - \frac{a_{13}a_{34}}{\sigma_{y_2}^2} + \frac{1}{\sigma_{y_1}^2} \left( \frac{a_{24}a_{14}a_{32} - a_{13}a_{24}^2}{\sigma_{y_2}^2} \right) \right]
\]

\[
\chi_{14} = \frac{1}{\sigma_{x_1}^2 \sigma_{y_2}^2} \left[ \frac{a_{14}}{\sigma_{y_1}^2} - \frac{a_{13}a_{34}}{\sigma_{x_2}^2} + \frac{1}{\sigma_{y_1}^2} \left( \frac{a_{13}a_{24}a_{32} - a_{14}a_{32}^2}{\sigma_{x_2}^2} \right) \right]
\]

\[
\chi_{23} = \frac{1}{\sigma_{y_1}^2 \sigma_{y_2}^2} \left[ \frac{a_{32}}{\sigma_{x_1}^2} + \frac{1}{\sigma_{x_1}^2} \left( a_{14}a_{13}a_{24} - a_{32}a_{14}^2 \right) - \frac{a_{24}a_{34}}{\sigma_{y_2}^2} \right]
\]

\[
\chi_{24} = \frac{1}{\sigma_{y_1}^2 \sigma_{y_2}^2} \left[ a_{24} + \frac{1}{\sigma_{x_1}^2 \sigma_{x_2}^2} \left( a_{13}a_{14}a_{32} - a_{24}a_{14}^2 \right) - \frac{a_{32}a_{34}}{\sigma_{x_2}^2} \right] \quad \text{(B-2)}
\]
Appendix C

The relation between the LCR $N_{\Xi}(r)$ of the channel envelope and $N_C(c)$ of the channel capacity

This part explains the equation (6.43) in chapter 6. The normalized time varying capacity process $C(t)$ is related to the channel fading process $\Xi(t)$ according to [Foschini 1998]

$$C(t) = \log_2 \left(1 + \frac{\gamma}{\gamma} \Xi^2(t)\right).$$ (C-1)

The LCR $N_C(c)$ of the channel capacity $C(t)$ is defined by

$$N_C(c) = \int_0^\infty \dot{c} p_{CC'}(c, \dot{c}) d\dot{c}.$$ (C-2)

From the transformation of random variables [Papoulis 2002, p.130] using $z = (2^c - 1)/\gamma$, $\dot{z} = (2^c \ln(2))/\gamma$, and with the corresponding Jacobian determinant $J = (2^c \ln(2)/\gamma)^2$, the joint PDF $p_{CC'}(c, \dot{c})$ can be written as

$$p_{CC'}(c, \dot{c}) = \left(\frac{2^c \ln(2)}{\gamma}\right)^2 p_{\Xi\dot{\Xi}} \left(\frac{2^c - 1}{\gamma}, \frac{2^c \dot{c} \ln(2)}{\gamma}\right).$$ (C-3)

Again, using the transformation of random variables $x = \sqrt{z}$, $\dot{x} = \dot{z}/(2\sqrt{z})$, and with the corresponding Jacobian determinant given by $J = 1/(4z)$, this joint PDF can be expressed as a function of the joint PDF $p_{\Xi\dot{\Xi}}(z, \dot{z})$ of the processes $\Xi(t)$ and $\dot{\Xi}(t)$ according to

$$p_{\Xi\dot{\Xi}}(z, \dot{z}) = \frac{1}{4z} p_{\Xi\dot{\Xi}} \left(\sqrt{z}, \frac{\dot{z}}{2\sqrt{z}}\right).$$ (C-4)
Now, substituting (C-4) in (C-3), and then (C-3) in (C-2) yields the following expression for the LCR $N_C(c)$ of the capacity channel $C(t)$

$$N_C(c) = \int_0^\infty \hat{c} \left( \frac{2^c \ln(2)}{\hat{\gamma}} \right)^2 \hat{\gamma} \frac{\hat{\gamma} - 1}{4(2^c - 1)} p_{\hat{\gamma}} \left( \sqrt{\frac{2^c - 1}{\hat{\gamma}}}, \frac{2^c \hat{\gamma} \ln(2)}{2\hat{\gamma} \sqrt{2^c - 1}} \right) \, dc. \quad (C-5)$$

Then, by letting $\hat{X} = 2^c \hat{\gamma} \ln(2) \sqrt{\frac{2^c - 1}{\hat{\gamma}}}$, (C-5) can be expressed as

$$N_C(c) = \int_0^\infty \hat{X} p_{\hat{\gamma}} \left( \sqrt{\frac{2^c - 1}{\hat{\gamma}}}, \hat{X} \right) \, d\hat{X}. \quad (C-6)$$

Finally, using the definition of the LCR $N_\Xi(r)$ [Rice 1944, Rice 1945], $N_C(c)$ can be written as

$$N_C(c) = N_\Xi \left( \sqrt{(2^c - 1)/\hat{\gamma}} \right). \quad (C-7)$$

It should be noted that the above property is generic and can be applied for all multipath fading models.


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